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ENGINEERING & MEDICAL COACHING

Test Solution

Booster Engineering

Test Code : PT04-1617-BE

Answer & Solution

Mathematics

1. (B)

Sol. Given that,

$$f(x)=\sqrt[3]{1-3x}+3\cos^{-1}\left(\frac{2x-1}{3}\right)+e^{3\tan x}$$

is defined, if $-1\leq\frac{2x-1}{3}\leq 1$

$\Rightarrow\quad -3\leq 2x-1\leq 3$

$\Rightarrow\quad -1\leq x\leq 2$

Domain of $f(x)=[-1,2]$

2. (C)

Sol. $\lim\limits_{x\to\alpha}\left(1+ax^2+bx+c\right)^{1/(x-\alpha)}$

$= \quad e^{\lim\limits_{x\to\alpha}}\frac{1}{(x-\alpha)}[(1+ax^2+bx+c)-1]$

$= \quad e^{\lim\limits_{x\to\alpha}}\frac{a(x-\alpha)(x-\beta)}{(x-\alpha)}=e^{a(\alpha-\beta)}$

3. (A)

Sol. Given that, $y=f(x^3)\Rightarrow\frac{dy}{dx}=f'(x^3).3x^2$

$\Rightarrow\quad \frac{dy}{dx}=3x^2\tan x^2$ [given, $f'(x)=\tan x$]

Also $\quad z=g(x^5)$

$\Rightarrow\quad \frac{dz}{dx}=g'(x^5)\cdot 5x^4=5x^4\sec x^5$

$\therefore\quad \frac{dy}{dx}=\frac{dydx}{dzdx}=\frac{3x^2\tan x^3}{5x^4\sec x^5}$

$=\frac{3}{5x^2}\cdot\frac{\tan x^3}{\sec x^5}$

4. (C)

Sol. $z=-2+2\sqrt{3}i=4\omega$

$z^{2n}+2^{2n}.z^n+2^{4n}=4^{2n}\cdot\omega^{2n}+2^{2n}\cdot 4^n\cdot\omega^n+2^{4n}$

$= \quad 4^{2n}[\omega^{2n}+\omega^n+1]$

$$=\begin{cases}0, \text{ if } n \text{ is not a multiple of 3.}\\ 3\cdot 4^{2n}, \text{ if } n \text{ is a multiple of 3.}\end{cases}$$

5. (C)

Sol. Given, mean, np = 2

and variance, npq = 1

$\Rightarrow\quad q=\frac{1}{2}$

$\therefore\quad p=1-q=\frac{1}{2}$

From Eq. (i), $\quad n\times\frac{1}{2}=2$

$\Rightarrow\quad$ n = 4

Now, $p(x>1)=p(x=2)+p(x=3)+p(x=4)$

$= \quad {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2+{}^4C_3\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2+{}^4C_4\left(\frac{1}{2}\right)^4$

$= \quad \frac{6+4+1}{16}$

$= \quad \frac{11}{6}$

6. (A)

Sol. Given that, $\quad xy=4$

$\Rightarrow\quad x\cdot\frac{dy}{dx}+y\cdot 1=0$

$\Rightarrow\quad \frac{dy}{dx}=\frac{y}{x}$

$\Rightarrow\quad \frac{dy}{dx}=-\frac{4}{x^2}\qquad [\because xy=4]$

Slope of the line $ax+by+c=0$ is $-\frac{a}{b}$

Since the given line is a tangent to the curve.

$\therefore\quad -\frac{4}{x^2}=-\frac{a}{b}\Rightarrow\frac{a}{b}>0$

Which is possible only when a > 0, b > 0 or a < 0, b < 0.

7. (C)

Sol. Let R be the set of families having a radio and T be the set of families having a TV.

$\therefore$ $n(R \cup T) = 1003 - 63 = 940,$

$n(R) = 794$

and $n(T) = 187$

Let $n(R \cap T) = x$

From Venn-diagram,

$794 - x + x + 187 - x = 940$

$\Rightarrow$ $981 - x = 940$

$\therefore$ $x = 41$

8. (B)

Sol. As shown in the figure, 1, 2 and X are the three boys and 3, 4 and Y are the three girls. Boy X will have neighbor as boys 1 and 2 and the girl Y will have neighbor as girls 3 and 4. Boys 1 and 2 can be arranged in P(2, 2) ways.

= 2! = 2 ways

Also, girls 3 and 4 can be arranged in P(2, 2) Ways

= 2! = 2 ways

Hence, required number of permutations

$= 2 \times 2 = 4$

9. (C)

Sol. Given system of linear equations has a non-zero solution, then $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix}$

Applying $R_2 \to R_2 - R_1 \to R_3 - R_1$, we get

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow$ $(3b-2a)(c-a) - (4c-2a)(b-a) = 0$

$\Rightarrow$ $3bc - 3ba - 2ac + 2a^2$

$= 4bc - 2ab - 4ac + 2a^2$

$\Rightarrow$ $2ac = bc + ab$

$\Rightarrow$ $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

Hence, a, b and c are in HP.

10. (D)

Sol. Let $AC = CB = hm$ and $\angle APC = \alpha$

Given that, $\frac{AP}{AB} = \frac{2}{1}$

$\Rightarrow$ $AP = 2AB = 4h$

In ΔAPC, $\tan\alpha = \frac{h}{4h} \Rightarrow \tan\alpha = \frac{1}{4}$

In ΔAPB, $\tan(\alpha+\beta) = \frac{2h}{4h} = \frac{1}{2}$

$\Rightarrow$ $\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = \frac{1}{2}$

$\Rightarrow$ $\frac{\frac{1}{4} + \tan\beta}{1 - \frac{1}{4}\tan\beta} = \frac{1}{2}$

$\Rightarrow$ $\frac{1 + 4\tan\beta}{4 - \tan\beta} = \frac{1}{2}$

$\Rightarrow$ $2 + 8\tan\beta = 4 - \tan\beta$

$\Rightarrow$ $9\tan\beta = 2$

$\therefore$ $\beta = \tan^{-1}\left(\frac{2}{9}\right)$

11. (B)

Sol. $\because \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\because$ $-1 \le \sin\theta \le 1$

Now, we take

$\log_{\sin\theta} \cos 2\theta = 2$

$\Rightarrow$ $\cos 2\theta = \sin^2\theta$

$\Rightarrow$ $1 - 2\sin^2\theta = \sin^2\theta$

$\Rightarrow$ $3\sin^2\theta = 1$

$\Rightarrow$ $\sin\theta = \pm\frac{1}{\sqrt{3}}$

$\therefore$ $\sin\theta = \frac{1}{\sqrt{3}}$

[since, base of any log cannot be negative]

Hence, given equation has a unique solution.

12. (C)

Sol. Since, $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

On putting $\theta = \dfrac{\pi}{9}$, we get

$$\Rightarrow \quad \tan\frac{\pi}{3} = \frac{3\tan\frac{\pi}{9} - \tan^3\frac{\pi}{9}}{1-3\tan^2\frac{\pi}{9}}$$

$$\Rightarrow \quad 3\left(1-\tan^2\frac{\pi}{9}\right)^2 = \left(3\tan\frac{\pi}{9} - \tan^3\frac{\pi}{9}\right)^2$$

$$\Rightarrow \quad \tan^6\frac{\pi}{9} - 33\tan^4\frac{\pi}{9} + 27\tan^2\frac{\pi}{9} = 3$$

13. (C)

Sol. Slope of line BS is $m_1 = \dfrac{b-0}{0-ae} = -\dfrac{b}{ae}$

and slope of line BS is $m_2 = \dfrac{b-0}{0+ae} = \dfrac{b}{ae}$

$\because \quad \angle SBS' = 90^{o}$

$\therefore \quad m_1 m_2 = -1$

$\Rightarrow \quad -\dfrac{b}{ae} \times \dfrac{b}{ae} = -1$

$\Rightarrow \quad b^2 = a^2e^2$

$\Rightarrow \quad a^2(1-e^2) = a^2 e^2$

$\Rightarrow \quad 1-e^2 = e^2$

$\Rightarrow \quad 2e^2 = 1$

$\Rightarrow \quad e = \dfrac{1}{\sqrt{2}}$

[since, e cannot be negative]

14. (C)

Sol. The equation of the tangent at $P(at^2, 2at)$ to $y^2 = 4ax$ is

$$ty = x + at^2 \qquad \textbf{...(i)}$$

It meets the directtrix $x = -a$.

$\therefore \quad ty = -a + at^2$

$\Rightarrow \quad y = \dfrac{a(t^2-1)}{t}$

Thus, Eq. (i) meets the directrix at $Q\left[-a, \dfrac{a(t^2-1)}{t}\right]$

Now, slope of PS is $m_1 = \dfrac{2at-0}{at^2-a} = \dfrac{2t}{t^2-1}$

and slope of QS is

$$m_2 = \frac{\frac{a(t^2-1)}{t} - 0}{-a-a} = \frac{(t^2-1)}{2t}$$

Now, $m_1 m_2 = \dfrac{2t}{t^1-1} \times \left(-\dfrac{t^2-1}{2t}\right) = -1$

$\therefore \quad \theta = \dfrac{\pi}{2}$

15. (D)

Sol. Let the equations of circle be

$$s_1 = x^2 + y^2 - 4x - 4y + 4 = 0$$

and $\quad s_2 \equiv x^2 + y^2 - x - y + \dfrac{1}{4} = 0$

Equation of common chord AB is

$$S_1 - S_2 = 0$$

$\Rightarrow \quad S_1 - S_2 = 0 \qquad \ldots.(i)$

Now, $\quad C_1M = \dfrac{4(2)+4(2)-5}{\sqrt{4^2+4^2}} = \dfrac{11}{4\sqrt{2}}$

and $\quad r_1 = \sqrt{4+4-4} = 2$

Also, equation of line C_1C_2 is

$$y = x \qquad \text{.... (ii)}$$

On solving Eqs. (i) and (ii), we get

$$x = \frac{5}{8} \text{ and } y = \frac{5}{8}$$

$\therefore$ Radius of required circle,

$$AM = \sqrt{AC_1^2 - C_1M^2}$$

$$= \sqrt{4 - \frac{121}{32}}$$

$$= \sqrt{\frac{7}{32}} = \frac{\sqrt{7}}{4\sqrt{2}}$$

Hence, required equation of circle is

$$\left(x - \frac{5}{8}\right)^2 + \left(y - \frac{5}{8}\right)^2 = \frac{7}{32}$$

16. (A)

Sol. $\sim p \vee q$ mean F $\vee$ F=F and $\sim$ r means F.

Now, $(\sim p \vee q) \wedge \sim$ r means F.

$\therefore$ $[(\sim p \vee q) \wedge \sim \text{r}] \Rightarrow$ p means T.

17. (C)

Sol. Let A and B denote the set of Americans who like cheese and apple, respectively

$\therefore$ $n(\text{A}) = 63, \ \text{n(B)} = 76$

we know that, $n(\text{A} \cup \text{B}) = \text{n(A)} + \text{n(B)} - \text{n}(\text{A} \cap \text{B})$

$\Rightarrow$ $n(\text{A} \cup \text{B}) = 63 + 76 - \text{n}(\text{A} \cap \text{B})$

$\Rightarrow$ $n(\text{A} \cap \text{B}) = 139 - \text{n}(\text{A} \cap \text{B})$

But $n(\text{A} \cup \text{B}) \geq 100$

$\Rightarrow$ $-n(\text{A} \cup \text{B}) \geq -100$

$\Rightarrow$ $139 - n(\text{A} \cup \text{B}) \geq 139 - 100 = 39$

$\Rightarrow$ $39 \geq n(\text{A} \cap \text{B})$ .....(i)

Again, $A \cap B \subseteq$ and $A \cap B \subseteq B$

$\therefore$ $n(\text{A} \cap \text{B}) \leq \text{n(A)} = 63$

and $n(\text{A} \cap \text{B}) \leq \text{n}(B) = 76$

$\therefore n(\text{A} \cap \text{B}) \leq 63$ ....(ii)

Then, $39 \leq n(\text{A} \cap \text{B}) \leq 63$

[from Equs. (i) and (ii)]

$\therefore$ $39 \leq x \leq 63$

18. (B)

Sol. **Case I** If $\log_3 x \geq 0 \Rightarrow x \geq 1$

From given equation $2(\log_3 \text{x})^2 = -\log_3 x + a = 0$

For real solution $(-1)^2 - 4 \cdot 2 \cdot a > 0$

$\therefore$ $a > \frac{1}{8}$ ...(i)

Case II If $\log_3 x < 0$

$\therefore$ $x > 1$

From given equation $2(\log_3 \text{x})^2 + \log_3 x + a = 0$

For real solution $(1)^2 - 4 \cdot 2 \cdot a > 0$

$\therefore$ $a < \frac{1}{8}$ ...(ii)

From equation

$$\log_3 x = \frac{-1 \pm \sqrt{1 - 8a}}{4} \quad [\text{here, } \log_3 x < 0]$$

$\Rightarrow$ $\frac{-1 \pm \sqrt{1 - 8a}}{4} < 0 \Rightarrow -1 \pm \sqrt{1 - 8a} < 0$

$\Rightarrow$ $\sqrt{1 - 8a} < 1$ [taking, +ve sign]

$\therefore$ a > 0 ...(iii)

Hence, from Eqs. (i), (ii) and (iii), we get $0 > a > \frac{1}{8}$

19. (D)

Sol. Given that $\lim_{x \to \infty} \left(\frac{x^2 + 1}{x + 1} - ax - b \right) = 0$

$$\Rightarrow \lim_{x \to \infty} \frac{(x^2 + 1) - (ax + b)(x + 1)}{x + 1} = 0$$

$$\Rightarrow \lim_{x \to \infty} \frac{x^2(1 - a) - (a + b)x - b + 1}{x + 1} = 0$$

$$\Rightarrow \lim_{x \to \infty} \frac{x(1 - a) - (a + b) - \frac{(b - 1)}{x}}{1 + \frac{1}{x}} = 0$$

which is possible, if

1 – a = 0 and a + b = 0

$\Rightarrow$ a = 1 and b = – 1

20. (D)

Sol. Since, $2\cos^2 x|\cos x|$ and $1-\cos^2 x$ are in GP.

$\therefore$ $\cos^2 x = 2\cos x(1-3\cos^2 x)$

$\Rightarrow$ $\cos x(6\cos^2 x+\cos x-2)=0$

$\Rightarrow$ $\cos x = 0$

and $(2\cos x-1)(3\cos x+2)=0$

$\Rightarrow$ $\cos x = 0, \frac{1}{2}$ and $-\frac{2}{3}$

21. (A)

Sol. $\because$ $f(x)=x^2+\frac{1}{x^2+1}$

$\therefore$ $f(x)=x^2+\left(\frac{1}{x^2+1}-1\right)+1$

$= (x^2+1)-\left(\frac{x^2}{x^2+1}\right)$

$= 1+x^2\left(1-\frac{1}{x^2+1}\right)$

$\geq 1, \forall x \in R$

Hence, range of $f(x)$ is $[1,\infty)$.

22. (C)

Sol. Let the equation of circle be

$x^2+y^2+2gx+2fy+c=0$

Since, this passes through (1, 2)

$\therefore$ $1^2+2^2+2g(1)+2f(2)+c=0$

$\Rightarrow$ $5+2g+4f+c=0$ .....(i)

Also, the circle $x^2+y^2=4$ intersects the circle $x^2+y^2+2gx+2fy+c=0$ orthogonally.

$\therefore$ On putting the value of c in Eq. (i), we get

$2g+4f+9=0$

Hence, the locus of centre (–g, –f) is

$-2x-4y+9=0$

or $2x+4y-9=0$

23. (B)

Sol. Let ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ [a > b]

$\therefore$ Area of ellipse = π ab

$= \pi a^2\sqrt{1-e^2}$

$= \pi a^2\sqrt{1-\frac{8}{9}} = \frac{\pi a^2}{3}$ $\left[\because e=\frac{2\sqrt{2}}{3}\right]$

Area of circle = πa^2

$\therefore$ Required probability = $\frac{\pi a^2-\frac{\pi a^2}{3}}{\pi a^2}=\frac{2}{3}$

24. (C)

Sol. Given equation can be rewritten as $(y-2)^2=12x$

Here, vertex and foci are (0, 2) and (3, 2).

So, vertex of the required parabola is (3, 2) and focus is (3, 4). The axis of symmetry is x = 3 and latusrectum = $4\cdot 2=8$

Hence, required equation is

$(x-3)^2=8(y-2)$

$\Rightarrow$ $x^2-6x-8y+25=0$

25. (A)

Sol. Let $B(x_1,y_1)$ and $C(x_2,y_2)$ be two vertices and $P\left(\frac{x_1+1}{2},\frac{y_1-2}{2}\right)$ lies on perpendicular bisector

$x-y+5=0$

$\therefore$ $\frac{x_1+1}{2}-\frac{y_1-2}{2}=-5$

$\Rightarrow$ $x_1-y_1=-13$

Also, PN is perpendicular to AB.

$\therefore$ $\frac{y_1+2}{x_1-1}\times 1=-1$

$\Rightarrow$ $y_1+2=-x_1+1$

$\Rightarrow$ $x_1+y_1=-1$

On solving Eqs. (i) and (ii), we get

$x_1=-7,\ y_1=6$

So, the coordinates of B are (–7, 6). Similarly, the coordinate of C are $\left(\frac{11}{5},\frac{2}{5}\right)$.

Hence, equation of BC is

$$y-6=\frac{\frac{2}{5}-6}{\frac{11}{5}+7}(x+7)$$

$\Rightarrow \quad y-6=-\frac{14}{23}(x-7)$

$\Rightarrow \quad 14x+23y-40=0$

26. (B)

Sol. Since, $\cos\theta=-\frac{\sqrt{3}}{2}<0$ and θ does not lie in third quadrant.

So, θ must be lying in 2nd quadrant.

$\Rightarrow \quad \tan\theta=-\frac{1}{\sqrt{3}}$

and $\quad \cos\theta=-\sqrt{3}$

Also, α lies in 3rd quadrant and $\sin\alpha=-\frac{3}{5}$

$\therefore \quad \tan\alpha=\frac{3}{4}$

and $\quad \cos\alpha=-\frac{4}{5}$

$$\therefore \quad \frac{2\tan\alpha+\sqrt{3}\tan\theta}{\cot^2\theta+\cos\alpha}=\frac{2\cdot\frac{3}{4}-\sqrt{3}\cdot\frac{1}{\sqrt{3}}}{3-\frac{4}{5}}$$

$$=\frac{\frac{3}{2}-1}{3-\frac{4}{5}}=\frac{5}{22}$$

27. (C)

Sol. Given that $\log_{\cos x}\sin x+\log_{\sin x}\cos x=2$

Let $\quad \log_{\cos x}\sin x=y$

$\therefore \quad y+\frac{1}{y}=2$

$\Rightarrow \quad (y-1)^2=0$

$\Rightarrow \quad y=1$

$\Rightarrow \quad \log_{\cos x}\sin x=1$

$\Rightarrow \quad \sin x=\cos x$

$\Rightarrow \quad \tan x=1$

$\Rightarrow \quad x=\frac{\pi}{4}$

28. (B)

Sol. Let $f(x)=4x^2-2x+a$

Since, both roots roots of $f(x)=0$ are lie in the interval (–1, 1)

$D\geq 0,\ f(-1)>0$ and $f(1)>0$

Consider $D\geq 0$,

$(-2)^2-4\cdot 4\cdot a\geq 0\Rightarrow a\leq\frac{1}{4}$ …(i)

Consider $f(-1)>0$,

$4(-1)^2-2(-1)+a>0\Rightarrow a>-6$ …(ii)

Consider $f(1)>0$,

$4(1)^2-2(1)+a>0\Rightarrow a>-2$ ….(iii)

From Eqs. (i), (ii) and (iii), we get $-2<a\leq\frac{1}{4}$

Hence, option (b) is correct.

29. (C)

Sol. **Statement I**

Since, $\cos(\sin x)=\sin(\cos x)$

Consider $\cos(\sin x)=\cos\left(\frac{\pi}{2}-\cos x\right)$

$\Rightarrow \quad \sin x=2n\pi\pm\left(\frac{\pi}{2}-\cos x\right), n\in I$

$\Rightarrow \quad \sin x\pm\cos x=\left(2n\pm\frac{1}{2}\right)\pi$

On squaring both sides, we get

$1\pm\sin 2x=\left(2n\pm\frac{1}{2}\right)^2\pi^2$

$\Rightarrow \quad |\sin 2x|=\left(2n\pm\frac{1}{2}\right)^2\pi^2-1$

But $\quad \left(2n\pm\frac{1}{2}\right)^2\pi^2>2$ for all $n\in I$

So, $|\sin 2x|>1$ which is not possible

Hence, the given equation does not possess real roots.

Statement II Since, $\sin x>0$

$\Rightarrow \quad 2n\pi<x<(2n+1)\pi,\ n\in I$

Hence, option (C) is correct.

30. (B)

Sol. $\sim[p\vee(\sim p\vee q)]=\sim p\wedge\sim(\sim p\vee q)$

$\equiv\sim p\wedge[\sim(\sim p)\wedge\sim q]$

$\equiv\sim p\wedge(p\wedge\sim q)$

Physics

31. (B)

Sol. Here x^2 has the dimensions of L^2, $B = [L^2]$

Also $ML^2T^{-2} = \frac{AL^{1/2}}{L^2}$ or $A = ML^{7/2}T^{-2}$

$\therefore A \times B = ML^{11/2}T^{-2}$

32. (D)

Sol. $y = a\sin\omega t + bt + ct^2\cos\omega t$

Here $a = y; b = y/t; c = y/t^2$

$\therefore a \times b \times c = y \times y/t \times y/t^2 = (y/t)^3$

33. (D)

Sol. Here relative velocity of the train w.r.t. other train is $V–v$. Hence, $0 – (V – v)^2 = 2ax$

or $a = \frac{(V-v)^2}{2x}$ Minimum retardation = $\frac{(V-v)^2}{2x}$

34. (C)

Sol. Displacement = Area under graph

$= 2\times2+\frac{1}{2}(2+6)\times1+\frac{1}{2}\times1\times6$

$-\frac{1}{2}\times1\times6-1\times6+2\times6 = 10\text{ cm}$

35. (C)

Sol. Relative velocity of boat with respect to water is

$\vec{v}_b - \vec{v}_w = 3\hat{i}+4\hat{j}-(-3\hat{i}-4\hat{j}) = 6\hat{i}+8\hat{j}$

36. (A)

Sol. Since $R = 2H$

or $\frac{v^2\sin2\theta}{g} = 2\times\frac{v^2\sin^2\theta}{2g}$

or $2\sin\theta\cos\theta = \sin^2\theta$ or $\tan\theta = 2$

$R = \frac{v^2\sin2\theta}{g} = \frac{v^2 2\sin\theta\cos\theta}{g}$

$= \frac{2v^2}{g}\times\frac{2}{\sqrt5}\times\frac{1}{\sqrt5} = \frac{4v^2}{5g}$

37. (B)

Sol. Consider the F.B.D of balloon. Net force on balloon is zero, So

$\vec{F}_3 + \vec{F}_4 = 0$

$-\vec{F}_1 - \vec{F}_2 = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$

$\vec{F}_1$ and $\vec{F}_2$ can't be action reaction pair because they do not act between the same two bodies. Similarly for $\vec{F}_3$ and $\vec{F}_4$.

38. (B)

Sol. Here mg = 49 or m = 5 kg

Required reading

R = 5(9.8 – 5) = 24 N

39. (A)

Sol. The FBD of the block is as shown in the figure.

N = 80 cos 37° = 64N

So, $f_L = 0.2 \times 64 = 32$N

As 4g < 80 sin 37°, friction force will act downwards. Net applied force in upward direction (excluding friction force) is

80sin 37° – 40 = 48 – 40 = 8 N

As $F_{applied}$ in vertical direction is less than f_L, block will not move in vertical direction and value of static friction force is $f = 8$ N.

40. (C)

Sol. N = Mg – Fsin ϕ

$a = \frac{F\cos\phi - \mu(Mg - F\sin\phi)}{M}$

41. (D)

Sol. Velocity of a projectile at any instant of time (t) is

$V^2 = v_x^2 + v_y^2 = (u\cos\theta)^2 + \left(u\sin\theta - g\frac{x}{u\cos\theta}\right)^2$

$\therefore KE = \frac{1}{2}mu^2 - mgx\tan\theta + \frac{mg^2x^2}{u^2\cos^2\theta}$

The given equation represents the equation of a parabola.

42. (C)

Sol. Given $v = k\sqrt{x}$ or $\frac{dx}{dt} = k\sqrt{x}$ or

$x^{-\frac{1}{2}}dx = k\,dt$

Integrating both sides, we get

$\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = kt + C;$ assuming $x(0) = 0$

Therefore, C= 0

$2\sqrt{x} = kt = \frac{k^2t^2}{4}$ or $v = \frac{k^2t}{2}$

Therefore, work done,

ΔW = Increase in KE

$= \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}m\left[\frac{k^2t}{2}\right]^2 = \frac{1}{8}mk^4t^2$

43. (C)

Sol. C_1 is the centre of mass of cut portion and C_2 that of remaining portion we have to find x_2.

$x_1 = 14 - 10.5 = 3.5$ cm

Mass will be proportional to area. So mass of the whole disc is $M = k\pi\,(14)^2$

Mass of cut portion $m_1 = k\pi\,(10.5)^2$

Mass of the remaining portion $m_2 = M - m_1 = k\pi\left(14^2 - 10.5^2\right)$

$= k\pi(24.5)\times(3.5)$

Now, $m_1x_1 = m_2x_2$

$\Rightarrow x_2 = \frac{m_1x_1}{m_2} = \frac{k\pi(10.5)^2\times 3.5}{k\pi(24.5)\times 3.5} = 4.5\,cm$

44. (D)

Sol. $\Delta p = (p + ep) + (ep + e^2p)$

$+ (e^2p + e^3p) + ...$

$= p(1+e)\left[1 + e + e^2 + ...\right] = \frac{p(1+e)}{1-e}$

45. (D)

Sol. Moment of inertia about 2 :

$I_2 = 4\left(\frac{ml^2}{3}\sin^2 45^\circ\right) = \frac{2ml^2}{3}$

Apply perpendicular axis theorem,

$I_1 = I_2 + mh^2 = \frac{2ml^2}{3} + 4m\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{8}{3}ml^2$

46. (D)

Sol. In case of rolling in the inclined plane, friction is static and acts in the upward direction and is given by

$$f = \frac{mg\sin\theta}{1 + \frac{R^2}{k^2}} \quad \text{.......... (i)}$$

For sphere, $k^2 = \frac{2}{5}R^2$ .... (ii)

From Eqs. (i) and (ii), $f = \frac{2}{7}mg\sin\theta$ (upwards)

47. (C)

Sol. Total energy, $E = \frac{1}{2}mv^2 - \frac{GmM}{r}$

$= -\frac{GmM}{2r} - \frac{Gmm}{r} = -G\frac{mM}{2r}$

$r = 2R = R = 3R$

$E = \frac{GmM}{6R}$

Potential energy $= -\frac{1}{6}\frac{GMm}{R} - \left(\frac{-GMm}{R}\right)$

$= \frac{5}{6}\frac{GMm}{R}$

$= \frac{5}{6}mgR$

48. (D)

Sol. $\vec{F} = \vec{F}_1 + \vec{F}_2$

As $|\vec{F}_1| = |\vec{F}_2|$

$\therefore |\vec{F}_1| = 2F_1\cos 30^\circ$

$= 2\frac{GM^2}{(2r)^2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}\frac{GM^2}{r^2}$

49. (D)

Sol. $Y = \frac{Fl}{a\Delta l}$, Y, l and F are constants.

$\therefore \Delta l \propto \frac{1}{D^2}$

$\frac{\Delta l_2}{\Delta l_1} = \frac{D_1^2}{D_2^2} = \frac{1}{6}$

$\therefore \Delta l_2 = \frac{1}{16} mm$

50. (C)

Sol. Energy stored/volume

$= \frac{1}{2} \times \text{stress} \times \text{strain}$

$= \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{\text{Y}} \quad \left(\text{Y} = \frac{\text{stress}}{\text{strain}} \right)$

$= \frac{(\text{stress})^2}{2Y} = \frac{S^2}{2Y}$

51. (C)

Sol. As $x = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2x}{r}}$

Velocity of efflux $v = \sqrt{2g(H - x)}$

Hence $R = vt = 2\sqrt{x(3H - x)}$

For range to be maximum $\frac{dR}{Dx} = 0$

Which gives $x = \frac{3}{2}H$.

Alternatively :

Maximum range is also equal to average of two heights

$x = \frac{H + 2H}{2} = \frac{3H}{2}$

52. (C)

Sol. $\rho A = 0.75, \rho_P = 0.6$

$\rho B = 1.0, \rho_Q = 0.9$

As relative density of P is lesser than B, so it will float in liquid B and as relative density of Q is greater than liquid A so it will sink, because if density of the object is greater than that of the liquid in which it is immerse, then its weight is more than the upthrust and vice versa.

53. (A)

Sol. $K_1 = 9K_2, l_1 = 18cm, l_2 = 6cm, \theta_1 = 100^\circ C,$

$\theta_2 = 0^\circ C$

Temperature of the junction

$$\theta = \frac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$$

$$\Rightarrow \theta = \frac{\frac{9K_2}{18}100 + \frac{K_2}{6} \times 0}{\frac{9K_2}{18} + \frac{K_2}{6}} = 75^\circ C$$

54. (B)

Sol. $\frac{\theta_1 - \theta_2}{t} \propto \left[\frac{\theta_1 + \theta_2}{2} - \theta \right]$

For the first condition

$\frac{60 - 50}{10} \propto \left[\frac{60 + 50}{2} - \theta \right]$

$\Rightarrow 1 + k[55 - \theta]$ ....(i)

For the second condition

$\frac{50 - 42}{10} \propto \left[\frac{50 + 52}{2} - \theta \right]$

$\Rightarrow 0.8 = K(46 - \theta)$ ....(ii)

From Eqs. (i) and (ii),

we get θ = 10°C

55. (C)

Sol. $W_{AB} = -P_0V_0$

$W_{BC} = 0$ and $W_{CD} = 4P_0V_0$

$\therefore W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

56. (D)

Sol. $1 \rightarrow 2$: isothermal, $\Delta U_{12} = 0$

$2 \rightarrow 3$: isothermic, $\Delta W = 0$

$\Rightarrow \Delta Q_{23} = \Delta U_{23} \Rightarrow -40 = \Delta U_{23}$

For a cyclic process, $\Delta U = 0$

$\Delta U_{12} + \Delta U_{23} + \Delta U_{31} = 0$

$0 + (-40) + \Delta U_{31} = 0$

$\Delta U_{31} = +40J$

57. (D)

Sol. $x = A \sin \omega t$

$$v = \frac{dx}{dt} = A\omega \cos \omega t = \omega\sqrt{A^2 - x^2}$$

$$a = \frac{dv}{dt} = -A\omega^2 \sin \omega t = \frac{dv}{dt} = -\omega^2 x$$

But $x = -\frac{a}{\omega^2}$

$$\therefore\ v = \omega\sqrt{\frac{A^2 - \frac{a^2}{\omega^4}}{}} \text{ or } v^2 = \omega^2\left(A^2 - \frac{a^2}{\omega^4}\right)$$

58. (D)

Sol. $y = 4\sin(10t + \phi),\ y_2 = 5\cos 10t$

$$v_1 = \frac{dy_1}{dt} = 40\cos(10t + \phi)$$

$$v_2 = \frac{dy_2}{dt} = -50\sin 10t = 50\cos\left(10t + \frac{\pi}{2}\right)$$

Phase difference between

v_1 and $v_2 = \left(\phi - \frac{\pi}{2}\right)$

59. (A)

Sol. Standard equation

$$y = A\sin(\omega t - kx + \phi_0)$$

In a given equation

$\omega = 7\pi,\ k = 0.04\pi$

$$v = \frac{\omega}{k} = \frac{7\pi}{0.04\pi} = 175\ m/s$$

60. (C)

Sol. $3 \times \frac{v}{4l_c} = 2 \times \frac{v}{2l_c}$ or $\frac{l_c}{l_0} = \frac{3}{4}$

Chemistry

61. A

62. C

63. B

64. C

Sol. $u = \sqrt{\frac{3RT}{M}}$

$$\frac{uH_2}{uN_2} = \sqrt{\frac{T(H_2)}{M(H_2)} \times \frac{M(N_2)}{T(N_2)}}$$

$$\sqrt{7} = \sqrt{\frac{T(H_2)}{T(N_2)} \times \frac{28}{2}}$$

$$7 = \frac{T(H_2)}{T(N_2)} \times 14$$

$7\ T(N_2) = 14\ T(H_2)$

$T\ (N_2) = 2 \cdot T(H_2)$

$\boxed{T(H_2) < T(N_2)}$

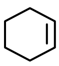
65. C

Sol. Aim

$$C + \frac{1}{2}O_2 \rightarrow CO$$

eq (i) $- \frac{1}{2}$ eq (ii) Gives the detired result

66. D

Sol.  $= -119.5$ kJ mol^{-1}

 $= -119.5$ kJ mol^{-1}

Actual enthalpy of Hydrogenation

$= -358 \cdot 5 - (-150 \cdot 4)$

$= -208 \cdot 1$ kJ.mol^{-1}.

67. C

68. C

Sol. $K = \frac{[HI]^2}{[H_2][I_2]}$

$[H_2] = [I_2] = 1$

$$\Rightarrow K = \frac{[HI]^2}{[I]^2}$$

$$= \frac{[HI]}{[I]} = \sqrt{K} = \sqrt{47 \cdot 6}$$

$= [HI] > [I_2]$.

69. D

70. D

Sol. From HCl

$[H^+] = 10^{-8}$ M

From H_2O

$[H^+] = 10^{-7}$ M

Total $= 10^{-8} + 10^{-7}$

$= 10–8\ (1 + 10)$

$= 11 \times 10^{-8}$

$pH = -\log [11 \times 10^{-8}]$

$pH = 6 \cdot 96$

71. A

Sol. $[HA] \rightleftharpoons H^+ + A^-$

$[H^+] = c \propto = 10^{-3}$

$0.1 \times \propto = 10^{-3}$

$\propto = 10^{-2}$

$\% \propto = 10\ \%$

72. C

73. C

74. D

Sol. Electron deficient

75. D

76. C

77. A

78. C

79. D

80. A

Sol. $CH \equiv CH + OH$ (OH) $\rightarrow$ $CH = CH$ (OH, OH)

81. B

82. B

83. D

84. C

85. C

86. C

87. A

88. B

89. D

90. C