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ENGINEERING & MEDICAL COACHING

Test Solution

Booster Engineering

Test Code : PT03-1617-BE

Answer & Solution

Mathematics

1. (A)

Sol. Put $\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}=\theta$

$\Rightarrow$ $\text{Cos}^{-1}\frac{\sqrt{5}}{3}=2\theta \Rightarrow \cos 2\theta=\frac{\sqrt{5}}{3}$

$\therefore$ $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{\sqrt{5}}{3}$

Using componend and dividend

$\frac{-1}{\tan^2\theta}=\frac{\sqrt{5}+3}{\sqrt{5}-3}$

$\Rightarrow$ $\tan^2\theta=\frac{3-\sqrt{5}}{3+\sqrt{5}}$

$\Rightarrow$ $\tan^2\theta=\frac{3-\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}=\frac{\left(3-\sqrt{5}\right)^2}{4}$

$\Rightarrow$ $\tan^2\theta=\frac{3-\sqrt{5}}{2}$

2. (B)

Sol. We have, $\tan^{-1}x+\cos^{-1}y=\tan^{-1}3, (x, y)\in N$

$\Rightarrow$ $\cos^{-1}y=\tan^{-1}3-\tan^{1}x$

$\Rightarrow$ $\tan^{1}\left(\frac{1}{y}\right)=\tan^{-1}\left(\frac{3-x}{1+3x}\right)$

$\Rightarrow$ $y=\frac{1+3x}{3-x}$

Hence, $x=1,2$, then y = 2, 7

$\therefore$ Solutions (1,2) and (2, 7).

3. (B)

Sol. Use $\cos^{-1}\frac{1-x^2}{1+x^2}$

= $\tan^{-1}\frac{2x}{1-x^2}$

4. (C)

Sol. Using –1, 0, 1 the Maximum value of determinant is : 2 and Min : –2

$\therefore$ The distinct value of 2×2 determinant = –2, –1, 0, 1, 2 i.e. 5.

5. (A)

Sol. $\cos(\tan^{-1}x)=\cos\theta$

where $\theta=\tan^{-1}x$ *i.e.* $\tan\theta=x$

$\therefore$ $\cos\theta=\frac{1}{\sqrt{1+x^2}}$

$\therefore$ $\cos(\tan^{-1}x)=\frac{1}{\sqrt{x+x^2}}$

$\cot^{1}[\cos(\tan^{-1}x]=\cot^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)=t$

$\Rightarrow$ $\cos t=\frac{1}{\sqrt{1+x^2}}$

$\therefore$ $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]=\frac{\sqrt{1+x^2}}{2+x^2}$

6. (B)

Sol. $\because$ $0\le[x]<2\Rightarrow[x]=0,1$

$-1\le[y]<1\Rightarrow[y]=-1,0$

and $1\le[z]<3\Rightarrow[z]=1,2$

Now, applying in the given determinate

$R_2\to R_2-R_1, R_3\to R_3-R_1$, then

$$\begin{vmatrix}[x] & [y] & [z]\\ -1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}$$

= ([x]+1) (1–0) – [y] (–1–0) + [z] (0+1)

= [x] + [y] + [z] +1

= 1 + 0 + 2 + 1 = 4

7. (B)

Sol. $T_n = \tan^{-1}\dfrac{1}{n^2+n+1} = \tan^{-1}\dfrac{1}{1+n(n+1)}$

$= \tan^{-1}\left[\dfrac{(n+1)-n}{1+(n+1)n}\right]$

$= \tan^{-1}(n+1) - \tan^{-1}(n)$

Now $T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$T_3 = \tan^{-1}4 - \tan^{-1}3$

....

....

$T_n = \tan^{-1}(n+1) - \tan^{-1}n$

$\therefore \quad T_1 + T_2 + ...T_n$

$= \tan^{-1}(n+1) - \tan^{-1}$

$= \tan^{-1}\dfrac{n+1-1}{1+(n+1)1}$

$= \tan^{-1}\dfrac{2}{n+2}$

Let $n \to \infty$

$\therefore \quad T_1 + T_2 + ...\infty$

$= \displaystyle\lim_{n\to\infty}\left[\tan^{-1}\frac{1}{1+\frac{2}{n}}\right] = \tan^{-1}1 = \frac{\pi}{4}$

Hence reqd. sum = $\dfrac{\pi}{4}$

8. (C)

Sol. Let $P = \cos^{-1}(\cos 12) - \sin^{-1}(\sin 12)$

$\because \quad \cos 12 > 0$ and $\sin 12 < 0$

$\therefore \quad P = \cos^{-1}\cos(4\pi - 12) - \sin^{-1}(12 - 4\pi)$

$= (4\pi - 12) - (12 - 4\pi) = 8\pi - 24$

9. (A)

Sol. For many sol : $\begin{vmatrix}\sin\theta & -1 \\ 1 & 2\end{vmatrix} = 0$

$\Rightarrow \quad 2\sin\theta + 1 = 0$

$\Rightarrow \quad \sin\theta = \dfrac{-1}{2}$

$\Rightarrow \quad \theta = \dfrac{-\pi}{2}, \dfrac{-5\pi}{6}.$

10. (C)

Sol. $\sin\dfrac{1}{2}\left[\cot^{-1}\left(\dfrac{-3}{4}\right)\right] = \sin\dfrac{1}{2}\left[\pi - \cot\cot^{-1}\dfrac{3}{4}\right]$

$= \sin\left[\dfrac{\pi}{2} - \dfrac{1}{2}\cot^{-1}\dfrac{3}{4}\right] = \cos\dfrac{1}{2}\cot^{-1}\dfrac{3}{4}$

$\cos\dfrac{1}{2}\cos^{-1}\dfrac{3}{5}$

Put $\dfrac{1}{2}\cos^{-1}\dfrac{3}{5} = \theta$

$\Rightarrow \quad \cos^{-1}\dfrac{3}{5} = 2\theta$

$\Rightarrow \quad \cos 2\theta = \dfrac{3}{5}$

$\Rightarrow \quad 2\cos^2\theta - 1 = \dfrac{3}{5}$

$\Rightarrow \quad 2\cos^2\theta = \dfrac{3}{5} + 1 = \dfrac{8}{5}$

$\Rightarrow \quad \cos^2\theta\dfrac{4}{5} \Rightarrow \boxed{\cos\theta = \dfrac{2}{\sqrt{5}}}.$

11. (A)

Sol. $A'B = [\,]_{4\times3}[\,]_{3\times...}$ and $BA' = [\,]_{3\times...}[\,]_{4\times3}$

$\therefore \quad$ order B is 3×4

12. (C)

Sol. As we know : $\sin^{-1} x + \cos^{-1} x = \pi/2$

$\therefore \quad 4\cos^{-1} x + \frac{\pi}{2} - \cos^{-1} x = \pi$

$\Rightarrow \quad 3\cos^{-1} x = \pi/2 \Rightarrow \cos^{-1} x = \pi/6$

$\Rightarrow \quad \boxed{x = \frac{\sqrt{3}}{2}}$.

13. (C)

Sol. $\alpha + \beta + \gamma = 0$

$\therefore \quad \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

As $R_1 + R_2 + R_3$.

14. (A)

Sol. $Cot^{-1}b - \cot^{-1} a - \cot^{-1} C$

$-\cot^{-1} b + \cot^{-1} a - \cot^{-} c$

15. ()

Sol. $A^2 + A - I = O$

$\Rightarrow \quad A^{-1}(A^2 + A - 1) = A^{-1} O$

$A + A^{-1}A - A^{-1} = O$

$(A + I) - A^{-1} = O \Rightarrow A^{-1} = A + I$

16. (D)

Sol. Multiple by its conjugate

$\cot^{-1}\left\{\frac{(1-\sin x)+(1+\sin x)+2\cos x}{(1-\sin x)-(1+\sin x)}\right\}$

$\cot^{-1}\left\{\frac{1+\cos x}{-\sin x}\right\}$

$\cot^{-1}\left\{\frac{2\cos^2 x/2}{-2\sin\frac{x}{2}\cos x/2}\right\} = \cot^{-1}\left\{-\cot\frac{x}{2}\right\}$

$\pi - \frac{\pi}{2}$

17. (A)

Sol. $\because \quad A^2 = 2A - A$

$A^3 = A^2.A = 2A^2 - I$

$A = 2A^2 - A = 2(2A - I) - A$

$= \quad 3A - 2I = 3A - (3-1)I$

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$A^n = nA - (n-1)I$.

18. (D)

Sol. $\tan(\cos^{-1} x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$

$\tan\cdot\tan^{-1}\frac{\sqrt{1-x^2}}{x} = \sin\ \sin^{-1}\frac{2}{\sqrt{5}}$

$\frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$

$\Rightarrow \quad \frac{1-x^2}{x^2} = \frac{4}{5}$

$\Rightarrow \quad 5 - 5x^2 = 4x^2$

$\Rightarrow \quad 9x^2 = 5$

$\Rightarrow \quad x^2 = \frac{5}{9}$

$\Rightarrow \quad x = \frac{\sqrt{5}}{3}$

19. (D)

Sol. $A^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{vmatrix}\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = I.$

20. (A)

Sol. $\tan^{-1}\frac{4x+6x}{1-24x^2} = \frac{\pi}{4}$

$\Rightarrow \quad \frac{10x}{1-24x^2} = 1$

$\Rightarrow \quad 1.24\pi x^2 = 10x$

$\Rightarrow \quad 24x^2 + 10x - 1 = 0$

$x = -\frac{1}{2} \quad or \quad \frac{1}{12}$

but $x=-\frac{1}{2}$ does not satisfy

21. (C)

Sol. Given : $\lambda^3=-2$

$$\lambda\begin{vmatrix}1 & 2\lambda & 1\\ \lambda^2 & 1 & 3\lambda^2\\ 2 & 2\lambda & 2\end{vmatrix}=(1-6\lambda^3)-2\lambda(\lambda^2-6\lambda^2)+(2\lambda^3-2)$$

$=\quad 1-6\times(-2)-2\times(-2)+12(-2)+2(-2)-2$

$=\quad 1+12+4-24-4-2=-13$

22. (A)

Sol. Holds.

23. (A)

Sol. $\cot^{-1}(\cos\alpha)^{\frac{1}{2}}+\tan^{-1}(\cos\alpha)^{\frac{1}{2}}=\frac{\pi}{2}$

$\cot^{-1}(\cos\alpha)^{\frac{1}{2}}-\tan^{-1}(\cos\alpha)^{\frac{1}{2}}=x$

$\Rightarrow\quad 2\tan^{-1}(\cos\alpha)^{\frac{1}{2}}=\frac{\pi}{2}-x$

$\tan^{-1}(\cos\alpha)^{\frac{1}{2}}=\frac{\pi}{4}-\frac{x}{2}$

$(\cos\alpha)^{\frac{1}{2}}=\tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right)=\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

$\cos\alpha=\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)^2$

$\cos\alpha=\frac{1+\tan^2\frac{x}{2}-2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}}$

$\frac{1-\cos\alpha}{1+\cos\alpha}=\frac{4\tan\frac{x}{2}}{2(1+\tan^2\frac{x}{2})}$

$\frac{1-\cos\alpha}{1+\cos\alpha}=\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\sin x$

$\frac{\alpha\sin^2\frac{\alpha}{2}}{\alpha\cos^2\frac{\alpha}{2}}=\sin$

$=\quad \sin x=\tan^2\frac{\alpha}{2}$

24. (B)

Sol. We have,

$=\quad 2\tan^{-1}(2\times1.414-1)$

$=\quad 2\tan^{-1}(1.828)>2\tan^{-1}\sqrt{3}=2\pi/3$

$\Rightarrow\quad A>2\pi/3$ .....(1)

Also, $B=3\sin^{-1}(1/3)+\sin^{-1}(3/5)$

$=\quad \sin^{-1}\left[3\times\frac{1}{3}-4\times\frac{1}{27}\right]+\sin^{-1}(3/5)$

$=\quad \sin^{-1}\left(\frac{23}{27}\right)+\sin^{-1}(0.6)$

$=\quad \sin^{-1}(0.8520)+\sin^{-1}(0.6)<\sin^{-1}(\sqrt{3}/2)$

$+\ \sin^{-1}(\sqrt{3}/2)$

$\Rightarrow\quad B<2\pi/3$ ...... (2)

From (1) and (2), we conclude A > B.

25. (C)

Sol. Easy Problem**,** Solve on your own**.**

26. (A, B)

Sol. $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$

$\Rightarrow\quad \frac{\pi}{2}-\cos^{-1}x+\frac{\pi}{2}-\cos^{-1}(1-x)$

$=\cos^{-1}x$

$\Rightarrow\quad 2\cos^{-1}x=\pi-\cos^{-1}(1-x)$

$\Rightarrow\quad \cos^{-1}(2x^2-1)=\cos^{-1}(x-1)$

$\Rightarrow\quad 2x^2-1=x-1$

$\Rightarrow\quad x(2x-1)=0$

$\Rightarrow\therefore\quad x=0,\frac{1}{2}$

27. (A, B, D)

Sol. For $\sin^{-1} x$ and $\cos^{-1} x$

Given $\frac{1}{2} < |x| < 1$ or $\frac{1}{2} < x < 1$

But $-1 < x < -\frac{1}{2}$ and $\tan^{-1} x, \ x \in R$

But for $\sec^{-1} x, x \geq 1$ and $x \leq -1$

28. (B, D)

Sol. $\because \ 6\sin^{-1}(x^2 - 6x + 8.5) = \pi$

$\therefore \quad \sin^{-1}(x^2 - 6x + 8.5) = \frac{\pi}{6}$

$\Rightarrow \quad x^2 - 6x + 8.5 = \sin\left(\frac{\pi}{6}\right)$

$\Rightarrow \quad x^2 - 6x + 8.5 = 0.5$

$\Rightarrow \quad x^2 - 6x + 8 = 0$

$\Rightarrow \quad (x-4)(x-2) = 0$

$\therefore \quad x = 2, 4$

29. (B, C)

Sol. $f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\sin\left(\frac{8\pi}{9}+\frac{\pi}{3}\right)} = e^{\cos^{-1}\sin\left(\frac{11\pi}{9}\right)}$

$= e^{\cos^{-1}\cos\frac{13\pi}{18}} = e^{\frac{13\pi}{18}}$

and $f\left(-\frac{7\pi}{4}\right) = e^{\cos^{-1}\sin\left(-\frac{7\pi}{4}+\frac{\pi}{3}\right)}$

$= e^{\cos^{-1}\sin\left(-\frac{17\pi}{12}\right)}$

$= e^{\cos^{-1}\cos\frac{\pi}{12}} = e^{\frac{\pi}{12}}$

30. (A, B, D)

Sol. $2\cot^{-1}7 = \tan^{-1}\left(\frac{1}{7}\right)$

$= \quad \cos^{-1}\left(\frac{1-\frac{1}{49}}{1+\frac{1}{49}}\right) = \cos^{-1}\frac{24}{25}$

Now, $2\cot^{-1}7 + \cos^{-1}\frac{3}{5}$

$= \quad \cos^{-1}\frac{24}{25} + \cos^{-1}\frac{3}{5}$

$= \quad \cos^{-1}\left(\frac{24}{25}\cdot\frac{3}{5} - \frac{7}{25}\cdot\frac{4}{5}\right) = \cos^{-1}\frac{44}{125}$

Since, $\frac{44}{125} > 0$

$\therefore \quad 0 < \cos^{-1}\frac{44}{125} < \frac{\pi}{2}$

Let $\cos^{-1}\frac{44}{125} = \theta$, then $\cos\theta = \frac{44}{125}$

$\therefore \quad \operatorname{cosec}\theta = \frac{125}{117}$ or $\theta = \operatorname{cosec}^{-1}\frac{125}{117}$

$\therefore \quad \theta = \cot^{-1}\frac{44}{117}$

Physics

31. (C)

Sol. Force on Q_2 is zero (q should be negative)

$$\frac{kQ_1Q_2}{R^2} = \frac{kQ_2}{x^2} \text{ or } \frac{x}{R} = \sqrt{\frac{q}{Q_1}}$$

Force on q is zero:

$$\frac{kQ_1q}{(R-x)^2} = \frac{kqQ_2}{x^2}$$

or $$\frac{R-x}{x} = \frac{\sqrt{Q_1}}{\sqrt{Q_2}}$$

or $$\frac{R}{x} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}} \text{ or } \frac{\sqrt{Q_1}}{\sqrt{q}} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}}$$

or $$\Rightarrow q = \frac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})}$$

32. (B)

Sol. $$\vec{r} = (9-3)\hat{i} + (12-4)\hat{j} = 6\hat{i} + 8\hat{j}$$

or $$r = \sqrt{6^2+8^2} = 10 \text{ m}$$

$$E = \frac{9\times10^9\times100\times10^{-6}}{10^2} = 9000 \text{ Vm}^{-1}$$

33. (B)

Sol. Charge on the element opposite to the gap is

$$dq = \frac{Q}{2\pi r}(0.002\pi)$$

$$= \frac{1}{2\pi(0.5)} \times \frac{2\pi}{1000} = 2\times10^{-3} \text{C}$$

$$E = \frac{9\times10^9\times2\times10^{-3}}{(0.5)^2} = 7.2\times10^7 \text{ NC}^{-1}$$

34. (B)

Sol. Force on the block: $F = qE$ toward left.

Let spring be compressed maximum by x. Then

$$Fx = \frac{1}{2}kx^2 \text{ or } qEx = \frac{1}{2}kx^2 \text{ or } x = \frac{2qE}{k}$$

35. (B)

Sol. $F_{net} = 2|F_{31}|\cos\alpha$

$$2\times\frac{1}{4\pi\varepsilon_0}\times\frac{2\times4\times10^{-12}}{(0.5)^2}\times\frac{4}{5} = 0.46 \text{ N}$$

36. (C)

Sol. As $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}-\vec{r}_0|^3}(\vec{r}-\vec{r}_0)$$

$$\overline{MN} = \vec{r}-\vec{r}_0 = (8\hat{i}-5\hat{j}) - (2\hat{i}+3\hat{j})$$

$$= (6\hat{i}-8\hat{j})$$

$$|\vec{r}-\vec{r}_0| = \sqrt{6^2+8^2} = 10 \text{ m}$$

$$\vec{E} = 9\times10^9\times\frac{50\times10^{-6}}{(10)^3}(6\hat{i}-8\hat{j})$$

$$\vec{E} = (2.7\hat{i}-3.6\hat{j})\text{k NC}^{-1}$$

37. (A)

Sol. Take PO as the x-axis and PA as the y-axis. Consider two elements EF and $E'F'$ of width $d\theta$ at angular distance θ above and below PO, respectively.

The magnitude of the field at P due to either element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{rd\theta\times Q(\pi\text{r}/2)}{r^2} = \frac{Q}{2\pi^2\varepsilon_0 r^2}dt$$

Resolving the fields, we find that the components along PO sum up to zero, and hence the resultant field is along PB. Therefore, field at P due to pair of elements is $2d\ E\sin\theta$

$$E = \int_0^{\pi/2} 2d\ E\sin\theta$$

$$= \int_0^{\pi/2}\frac{Q}{2\pi\varepsilon_0 r^2}\sin\theta d\theta = \frac{Q}{\pi^2\varepsilon_0 r^2}$$

38. (B)

Sol. Let us consider the electric field due to wire (3) only

$$\vec{E}_3 = E\hat{u}$$

$$\vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0(a^2+a^2)^{1/2}}(\hat{i}\cos 45^\circ + \hat{j}\cos 45^\circ)$$

$$= \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

$$\vec{E}_3 = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i}+\hat{j})$$

Similarly, electric field due to wires (1) and (2)

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{j}+\hat{k}) \text{ and } \vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i}+\hat{k})$$

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_{\text{net}} = \frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$$

39. (B)

Sol. $E_1 = E_4 = 2E_2 = 2E_3$

Horizontal components will be canceled, net field will be upward.

40. (C)

Sol. From the direction of fields from graph, it is clear that q_1 is negative and q_2 is positive. Now since electric field is zero to the right of q_2, q_2 should be smaller in magnitude.

41. (A)

Sol. $F_e = F_2 + F_3$

$$= \frac{kq^2}{(\text{L}\sin\theta)^2} + \frac{kq^2}{(2L\sin\theta)^2}$$

$$F_e = \frac{5}{4}\frac{kq^2}{L^2\sin^2\theta} \qquad \text{(i)}$$

$$T\sin\theta = \text{Fe} \qquad \text{(ii)}$$

$$T\cos\theta = mg \qquad \text{(iii)}$$

From (i), (ii), and (iii)

$$q = \sqrt{\frac{16}{5}\pi\varepsilon_0 mgL^2\sin^2\theta\tan\theta}$$

42. (D)

Sol. $\frac{kQ_2}{x^2} = \frac{kQ_1}{(x-R)^2}$ or $x = \frac{R}{2}$

43. (C)

Sol. For external points, a charged sphere behaves as if the whole of its charge is concentrated at its center.

Force on A due to B,

$$F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{ along } \overrightarrow{BA}$$

And force on A due to C,

$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{ along } \overrightarrow{CA}$$

Now as angle between BA and CA is 60° and

$$|F_{AB}| = |F_{AC}| = F$$

$$\therefore \quad F_A = \sqrt{F^2+F^2+2FF\cos 60} = \sqrt{3}F$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$$

44. (C)

Sol. The dielectric gets polarized as shown. Induced charges will also contribute in electric field.

So, intensity at points A and C will increase and at B intensity will decrease.

45. (A)

Sol. Electric field on surface of a uniformly charged sphere is given by

$$\frac{Q}{4\pi\varepsilon_0 R^2} = \frac{\rho R}{3\varepsilon_0}$$

Electric field at outside point is given by

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

$$E_B = E_{\text{whole sphere}} - E_{\text{cavity}}$$

$$= \frac{\rho r_0}{3\varepsilon_0} - \frac{\rho\left(\frac{r_0}{2}\right)^3}{3\varepsilon_0\left(\frac{3r_0}{2}\right)^2} = \frac{17\rho r_0}{54\varepsilon_0}$$

46. (B)

Sol. Let us complete the sphere. Electric field due to lower part at A is equal to electric field due to upper part at $B = E$ (given)

Electric field due to lower part at B = electric field due to upper part

$$= \frac{kQ}{(2R)^2} - E$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\rho(4/3)\pi R^3}{4R^2} - E$$

$$= \frac{\rho R}{12\varepsilon_0} - E$$

47. (C)

Sol. $\vec{E}_p = \frac{-\rho}{3\varepsilon_0}\vec{s} = -\frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{d})$

$$= \frac{\rho}{3\varepsilon_0}(\vec{d} - \vec{r})$$

48. (A)

Sol. $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R_2}$

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi F R^2}$$

$$\varepsilon_0 = \frac{C^2}{N.m^2} = \frac{[\text{AT}]^2}{MLT^{-2}\cdot L^2}$$

$$= [M^{-1}L^{-3}T^4A^2]$$

49. (C)

50. (D)

Sol. V is a scalar quantity, and E is a vector quantity

51. (D)

Sol. $AC = \sqrt{2}\,t = BD$

or $BO = \frac{1}{\sqrt{2}}$

$$F_{BO} = F_{BD} + (F_{BA} + F_{BC})\cos 45^\circ$$

Solving, we get

$Q = \frac{q}{4}(1 + 2\sqrt{2})Q$ should be negative of q.

52. (B)

Sol. $W_{AB} = W_{AC} + W_{CB}$

W_{CB} should be zero, because in moving from C to B, we always move perpendicular to field. Hence, force applied by field and displacement will be at 90°.

$$W_{AC} = -e(V_C - V_A)$$

$\therefore\ V_C - V_A = -E \times AC = -10 \times 4 = -40$

$$W_{AC} = -e \times (-40) = 40e$$

So $W_{AB} = 40e$ J = 40 eV

53. (A)

Sol. Net force $F_{net} = qE_x$

$$F = q\frac{\lambda}{4\pi\varepsilon_0} = \frac{\lambda q}{4\pi\varepsilon_0 r}$$

54. (D)

Sol. Four between two charges does not depend upon the presence or absence of third charge.

55. (B)

Sol. $U = \frac{kqQ}{r} - \frac{kqQ}{r} + \frac{kq^2}{2r} = 0$ or $Q/q = 1/4$

56. (D)

Sol. $E_x = -\frac{dV}{dx} = -4x = -4\times 2 = -8$

$E_y = 0, E_z = 0$

Hence, $\vec{E} = -8\hat{i}\ NC^{-1}$

57. (C)

Sol. From the free body diagram of the sphere, using Lami's theorem,

we get $\frac{F}{mg} = \tan\theta$ (i)

When suspended in liquid, as q remains same,

$$\frac{F'}{mg\left(1-\frac{\rho}{d}\right)} = \tan\theta \quad \text{(ii)}$$

Using (i) and (ii), we get

$$\frac{F}{mg} = \frac{F'}{mg\left(1-\frac{\rho}{d}\right)} \text{ where } F' = \frac{F}{K}$$

$$= \frac{F'}{mgK\left(1-\frac{\rho}{d}\right)}$$

$$\Rightarrow K = \frac{1}{1-\frac{\rho}{d}} = 2$$

58. (A)

Sol. $T\sin\theta = F = \frac{kq^2}{d^2},\ T\cos\theta = mg$

$$\Rightarrow \tan\theta = \frac{k}{mg}.\frac{q^2}{x^2} \Rightarrow \frac{x}{2\ell} = \frac{k}{mg}.\frac{q^2}{x^2}$$

$$\Rightarrow x^3 = \frac{2k\ell}{mg}q^2 \Rightarrow q \propto x^{3/2}$$

$$\Rightarrow \frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt} \quad \left(\frac{dq}{dt}\text{ is constant}\right)$$

$$\Rightarrow c \propto x^{1/2}v \Rightarrow v \propto x^{-1/2}$$

59. (C)

60. (C)

Sol. Linear charge density is

$$\lambda = \frac{q}{2\pi}$$

$$E = \int dE\sin\theta(-\hat{j}) = \int \frac{K\times dq}{r^2}\sin\theta(-\hat{j})$$

$$= \frac{K}{r^2}\int \frac{dr}{\pi r}d\theta\sin\theta(-\hat{j})$$

$$= \frac{K}{r^2}\frac{q}{\pi}\int_0^{\pi}\sin\theta(-\hat{j})$$

$$= \frac{-q}{2\pi^2\varepsilon_0 r^2}(-\hat{j})$$

Chemistry

61. A.

62. C.

Sol. $2(C_6H_5COOH) \longrightarrow (C_6H_5COOH)_2$

1   0

$1 - \alpha$ $\alpha/2$

Total = $1-\alpha+\frac{\alpha}{2}$

$i = \frac{1-\alpha+\frac{\alpha}{2}}{1}$

$i = 1-\alpha+\frac{\alpha}{2}$

$\boxed{i = 1-\frac{\alpha}{2}}$

63. B.

As $\boxed{\text{Volume} \propto T}$

64. C.

65. D

66. D

67. C

Sol. Density = 1·179/cc

∴ concⁿ = 1170 g/e^{-1}

$= \frac{1170}{36\cdot5} = 32\cdot05\ M.$

68. B

Sol. $6\cdot02 \times 10^{20}$ molecules = $\frac{6\cdot02\times10^{20}}{6\cdot022\times10^{23}}$ moles

$= 10^{-3}$ mol.

∴ molar concⁿ = $\frac{10^{-3}}{100}\times1000 = 0.01\ M.$

69. A

Sol. $n^2 = \frac{n_2}{n_1+n_2} = \frac{n_2}{\frac{w_1}{w_2}+n_2}$

If $w_1 = 1000\ g$

n_2 = molality (m)

$m_1 = 78.$

$\Rightarrow \quad \frac{m}{\frac{1000}{78}+m} = 0\cdot2$

$\Rightarrow \quad m = 3\cdot2.$

70. A

Sol. Concⁿ = of cone sugar = 5·12% = 51·2 gl^{-1}

$\frac{51\cdot2}{342}$ mol l^{-1}

Concⁿ = of unknown Solⁿ = 0·9 %

= 9 g/l

$= \frac{9}{m}$ mol l^{-1}

Solⁿ= are isotonic

$\Rightarrow \frac{9}{m} = \frac{51\cdot2}{342}$

$m = 60.$

71. A.

$2KI + HgI_2 \rightarrow K_2HgI_2$

Due to molecular association F.P. is raised.

72. B

Sol. $Ba(NO_3)_2 \rightleftharpoons Ba^{2+} + 2NO_3^-$

1 0 0

$1-\alpha$ α 2α

Total = $1 + 2\alpha$

$i = 1 + 2\alpha$

$\alpha = \frac{i-1}{2}$

$\alpha = \frac{2\cdot 74 - 1}{2}$

$\alpha = 0\cdot 87$

$\% \alpha = 87\%$

73. C

74. B

75. C

Sol. no. of sphere = 4

Hence vol = $4 \times \frac{4}{3}\pi r^3 = \frac{16}{3}\pi r^3$.

76. B

77. C

78. C

79. B

Sol. $2(C_6H_5COOH) \rightleftharpoons (C_6H_5COOH)_2$

80. A

Sol. $P_S = P_A^o x_A + P_B^o x_B$

$= 0\cdot 5 \times 120 + 0\cdot 5 \times 80$

$= 100 \Longrightarrow$ Ideal (Sol^n)

81. A

82. B.

Sol. $\frac{\Delta T_b}{\Delta T_f} = \frac{kb \cdot m}{kf \cdot m}$

$\frac{\Delta T_b}{0\cdot 186} = \frac{0\cdot 52}{1\cdot 86}$

$\Delta T_b = 0\cdot 52^\circ C$

$\therefore$ B. · P. = 100 + 0·052

= 100·052°C

83. A

84. C

85. A

Sol. $BaCl_2$ > NaCl > Glucose

$BaCl_2$: Three particle; NaCl: Two particle; Glucose ↓ undissosiated

86. A

87. C

88. D

89. A

90. D