eduVibrant
ENGINEERING & MEDICAL COACHING

# Test Solution

## Booster Engineering

Test Code : PT02-1617-BE

## Answer & Solution

## Mathematics

**1. (A)**

**Sol :** for $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ to be purely in.

e(z) = 0

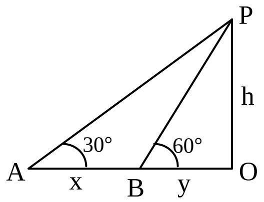
$\Rightarrow \frac{2+3i\sin\theta}{1-2i\sin\theta}\times\frac{1+2i\sin\theta}{1+2i\sin\theta}$

$\Rightarrow 2-6\sin^2\theta \qquad \Rightarrow \sin^2\theta = \frac{2}{3}$

$\Rightarrow \sin^2\theta = \frac{1}{3} \qquad \sin\theta = \frac{1}{\sqrt{3}}.$

**2. (A)**

**Sol :**




Given : 10 min to Reach A to B.

$\frac{h}{x+y} = \tan 30^\circ \Rightarrow h = \frac{1}{\sqrt{3}}(x+y) \quad \ldots(1)$

$\frac{h}{y} = \tan 60^\circ \Rightarrow h = \sqrt{3}y \quad \ldots(2)$

From (1) and (2)

$\frac{1}{\sqrt{3}}(x+y) = \sqrt{3}y$

$\Rightarrow x+y = 3y \Rightarrow \boxed{x = 2y}$

Let u be the speed. ∴ 10u = 2 ut ⇒ $\boxed{t = 5}$

**3. (C)**

**Sol.** S.d = 3.5

⇒ Van = $(S.d)^2 = (3.5)^2 = 12.25$

Van = $\frac{1}{2}\Sigma(zi - \overline{x})^2$

Data : 2, 3, $a$ and 11

$\overline{x} = \frac{2+3+a+11}{4} = \frac{16+a}{4}$

Van = $\frac{1}{4}\left\{\left(2-\frac{16+a}{4}\right)+\left(3-\frac{16+a}{4}\right)^2+\left(a-\frac{16+a}{4}\right)+\left(11-\frac{16+a}{4}\right)\right\}.$

$\Rightarrow 12.25 = \frac{1}{4}\left\{\frac{(a+8)^2+(a+4)^2+(3a-16)^2+(28-a)^2}{16}\right\}$

$\Rightarrow 12.25\times4\times16 = 12a^2+16a+8a-96a-56a+80+256+784$

$\Rightarrow 784 = 12a^2-128a+336+784$

$\Rightarrow 12a^2-128a+336 = 0$

$3a^2-32a+84 = 0$

**4. (D)**

**Sol.** len of LR = 8

dist b/w foci = 2ac

$\alpha\frac{1}{2}(2ac) = 2b.$

$= \frac{b}{a} = c/2$

Now, $e^e = 1+\frac{b^2}{a^2}$

$= e^e = \frac{e^2}{4}+1, \qquad 4e^e = e^2+4$

$3e^2 = 4 \Rightarrow e^2 4/3, \quad e^2 = 2/\sqrt{3}$

**5. (B)**

**Sol :** Both points must lie on normal

∴ Eq$^n$ of Normal to $y^2 = 4ax$

$y+tx = 2at+at^3$ have a = 2

∴ $y+tx = 4t+2t^3$

∴ x, y ≡ 0, –6 ∴ $-6 = 4t+2t^2$.

$t^3+2t+3 = 0 \qquad \Rightarrow$ t = –1

∴ Eq$^n$ of Normal

$y - x = -4 - 2$

$\Rightarrow \boxed{x - y = 6}$

For p we have to solve line & problem $y^2 = 8x$

$\therefore\ (x-6)^2 = 8x$

$\Rightarrow\ x^2 - 12x + 36 = 8x$

$\Rightarrow\ x^2 - 20x + 36 = 0$

$x = 2, 18$

For x = 2 ; 4 = –4

$\therefore$ Centre : (2, –4)

Red : $\sqrt{4+4} = 2\sqrt{2}$

$\therefore$ Eq$^n$ F c :

$(x-2)^2 + (y+4)^2 = 8$

$\Rightarrow x^2 + y^2 - 4x + 8y + 12 = 0;$

**6. (D)**

**Sol.** $\dfrac{x-y+1}{\sqrt{2}} = \pm\dfrac{7x-y-5}{5\sqrt{2}}$

= $x + 2y - 5 = 0$

= $2x - 4 = 0$

= $(x+1) + 2(4+2) = 0$

= $x + 2y + 5 = 0$

= $(x+1) - (y+2) = 0$

= $2x - y = 0$

**7. (A)**

**Sol.** $A \to \dfrac{41}{21} = 12$

$L \to 41 = 24$

$m \to \dfrac{41}{21} = 12$

$SA \to \dfrac{31}{2} = 3$

$SL\ \ 31 = 6$

SMALL = 58

**8. (C)**

**Sol :** $a_1, a_2 ............ a_9$ are in AP.

$\therefore\ a_2 = a_1 + d$

$a_5 = a_1 + 4d$

$a_9 = a_1 + 8d$

A/Q : $a_5^2 = a_1 + a_9$

$\Rightarrow (a_1 + 4d)^2 = (a_1 + d)(a_1 + 8d).$

$\Rightarrow ad^2 + 16d^2 + 8a_1 d$

$= a_1^2 + 8a_1 d + a_1 d + 8d^2$

$8d^2 = a_1 d \Rightarrow a_1 = 8d$

$\therefore\ a_2 = 9d\ ; a_5 = 12d;\ a_9 = 16d$

$\therefore\ \dfrac{a_5}{a_2} = \dfrac{12}{9} = 4/3$

**9. (C)**

**Sol :** $S = \dfrac{8^2}{5^2} + \dfrac{12^2}{5^2} + \dfrac{16^2}{5^2} + ....$

$= \dfrac{4^2}{5^2}\left(2^2 + 3^2 + ........ 11^2\right)$

Add n sub $1^2$

$= \dfrac{4^2}{5^2}\left(1^2 + 2^2 ........ 11^2 - 1^2\right)$

$= \dfrac{4^2}{5^2}\left(\dfrac{11\times(11+1)\times(2\times 11+1)}{6} - 6\right).$

$= \dfrac{16}{25}\left(\dfrac{11\times 12\times 23}{6} - 2\right).$

$= \dfrac{16}{25}(22\times 23 - 1).$

$S = \dfrac{16}{25}\times 505$

$S = \dfrac{16}{5}\times 101\ \therefore m = 101.$

**10. (D)**

**Sol.** $(A\cap B)\cup C = (A\cup B)\cap(B\cup C)$

$\therefore$ $(P\vee q)\vee(\vee q\vee q)\vee(\sim p\wedge q)$

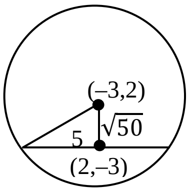
$\vee q\vee q$ is always true

$\therefore$ $(p\vee q)\vee(\vee p\wedge q)$

= $p\vee q$

(Draw Vann diagram)

**11. (C)**




**Sol :** $\therefore$ Eq$^n$ : $(x+3)^2+(y-2)^2=75$

$\therefore$ Red : $5\sqrt{3}$.

**12. (A)**

**Sol :** Let the centre be h, k.

$\therefore$ dist b/w centre = $R_1 + R_2$

$\therefore$ Centre of given circle

(4, 4) and $R_1 = 6$

And $R_2 = K$

$\therefore (h-4)^2+(\alpha-4)^2=(6+k)^2$.

$\Rightarrow h^2+k^2-8h-8k+32=k^2+12k+36$

Rep h by x n k by y

$x^2-8x-8y+32=12y+36$

$x^2-8x-4=20y$

Parabola

**13. (D)**

**Sol :** Let $p=\lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$.

Form is $1^\infty$

$\therefore p=e^{x\to 0^+\left(1+\tan^2\sqrt{x}-1\right)\cdot\frac{1}{x}}$

$= e^{x\to 0^+\frac{\tan^2\sqrt{x}}{2x}}$

$= e^{x\to 0^+\frac{\tan^2\sqrt{x}}{2\left(\sqrt{x}\right)^2}}$

$P=e^{\frac{1}{2}}$

$\log P=\frac{1}{2}$.

**14. (D)**

**Sol :** $\cos x+\cos 2x+\cos 3x+\cos 4x=0$

$2\cos 2x\cos x+2\cos 3x\cos x=0$

$\cos x\cdot 2\cos\frac{5x}{2}\cos\frac{x}{2}=0$

$\cos x\cdot\cos\frac{x}{2}\cdot\cos\frac{5x}{2}=0$

$x=\frac{\pi}{2},\frac{3\pi}{2}$.........(2)

$x=\pi$.........(1)

$\frac{5x}{2}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},\frac{9\pi}{2},$

$\frac{\pi}{5},\frac{3\pi}{5},\frac{2\pi}{5},\frac{9\pi}{5}$.

**15. (D)**

**Sol :** $F(x)+2F\left(\frac{1}{x}\right)=3x$.....(1)

Replacing $x$ by $\frac{1}{x}$

$F\left(\frac{1}{x}\right)+2F(x)=3/x$.....(2)

Eliminating $F\left(\frac{1}{x}\right)$ using

(1) & (2)

$3F(x)=\frac{6}{x}-3x\Rightarrow \boxed{F(x)=\frac{2}{x}-x}$

For $F(x)=\frac{2}{x}-x$.

$\Rightarrow\frac{2}{x}-x=-\frac{2}{x}+x$

$\Rightarrow\frac{4}{x}-2x=0$

$\Rightarrow 4-2x^2=0$

$\Rightarrow\boxed{x=\pm\sqrt{2}}$

**16. (C)**

**Sol.** $\left|\frac{z_1-2z_2}{2-z_1\overline{z}_2}\right|=1$

$\Rightarrow |z_1-2z_2|^2=|2-z_1\overline{z}_2|^2$

$\Rightarrow (z_1-2z_2)(\overline{z_1-2z_2})=(2-z_1\overline{z}_2)(\overline{2-z_1\overline{z}_2})$

$\Rightarrow (z_1-2z_2)(\overline{z}-2\overline{z}_2)=(2-z_1\overline{z}_2)(2-\overline{z}_1z_2)$

$\Rightarrow (z_1\overline{z}_1)-2z_1\overline{z}_2-2\overline{z}_1z_2+4z_1\overline{z}_2$

$\Rightarrow 4-2\overline{z}_1z_2-2z_1\overline{z}_2+z_1\overline{z}_1z_2\overline{z}_2$

$\Rightarrow |z_1|^2+4|z_2|^2=4+|z_1|^2|z_2|^2$

$\Rightarrow |z_1|^2+4|z_2|^2-4|z_1|^2|z_2|^2=0$

$\Rightarrow (|z_1|^2-4)(1-|z_2|^2)=0$

$\because |z_2|\neq 1$

$\therefore |z_1|^2=4$

$\Rightarrow |z_1|=2$

$\Rightarrow$ Point $z_1$ lies on circle of radius 2.

**17. (C)**

**Sol.** $\alpha,\beta=\dfrac{6\pm\sqrt{36+8}}{2}=3\pm\sqrt{11}$

$\alpha=3+\sqrt{11},\ \beta=3-\sqrt{11}$

$\therefore a_n=(3+\sqrt{11})^n-(3-\sqrt{11})^n$

$\dfrac{a_{10}-2a_8}{2a_9}$

$=\dfrac{(3+\sqrt{11})^{10}-(3-\sqrt{11})^{10}-2(3+\sqrt{11})^8+2(3-\sqrt{11})^8}{2\left[(3+\sqrt{11})^9-(3-\sqrt{11})^9\right]}$

$=\dfrac{(3+\sqrt{11})^8\left[(3+\sqrt{11})^2-2\right]+(3-\sqrt{11})^8\left[2-(3-\sqrt{11})^2\right]}{2\left[(3+\sqrt{11})^9-(3-\sqrt{11})^9\right]}$

$=\dfrac{(3+\sqrt{11})^8(9+11+6\sqrt{11}-2)+(3-\sqrt{11})^8(2-9-11+6\sqrt{11})}{2\left[(3+\sqrt{11})^9-(3-\sqrt{11})^9\right]}$

$=\dfrac{6(3+\sqrt{11})^9-6(3-\sqrt{11})^9}{2\left[(3+\sqrt{11})^9-(3-\sqrt{11})^9\right]}=\dfrac{6}{2}=3$

**18. (B)**

**Sol.** For digits number can be arranged in $3\times4!$ ways. Five digits number can be arranged in $5!$ ways. Number of integers $=3\times4!+5!=192$.

**19. (A)**

**Sol.** $(1-2\sqrt{x})^{50}={}^{50}C_0={}^{50}C_1 2\sqrt{x}+{}^{50}C_2(2\sqrt{2})$ ...(1)

$(1+2\sqrt{x})^{50}={}^{50}C_0={}^{50}C_1 2\sqrt{x}-{}^{50}C_2(2\sqrt{2})^2$

$+\ldots+{}^{50}C_3(2\sqrt{x})^3-{}^{50}C_4(2\sqrt{x})^4$ ...(2)

Adding equation (1) and (2)

$(1-2\sqrt{x})^{50}+(1+2\sqrt{x})^{50}$

$=2\left[{}^{50}C_0+{}^{50}C_2 2^2x+{}^{50}C_4 2^3x^2+\ldots\right]$

Putting x = 1, we get above as $\dfrac{3^{50}+1}{2}$

**20. (B)**

**Sol.** $m=\dfrac{l+n}{2}$ and common ratio of G.P. = $r=\left(\dfrac{n}{l}\right)^{\frac{1}{4}}$

$\therefore$ $G_1=l^{3/4}n^{1/4},G_2=l^{1/2}n^{1/2},G_3=l^{1/4}n^{3/4}$

$G_1^4+2G^4+G_3^4=l^3n+2l^2n^2+l\,n^3$

$=l\,n(l+n)^2$

$=l\,n\times2m^2$

$=4lm^2n$

**21. (B)**

**Sol.** $N^{th}$ term of series $=\dfrac{\left[\dfrac{n(n+1)}{2}\right]^2}{n^2}=\dfrac{1}{4}(n+1)^2$

Sum of n term $=\sum\dfrac{1}{4}(n+1)^2$

$= \frac{1}{4}\left[\sum n^2 + 2\sum n + n\right]$

$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n\right]$

Sum of 9 terms

$\frac{1}{4}\left[\frac{9\times10\times9}{6} + \frac{18\times10}{2} + 9\right] = \frac{384}{4} = 96$

**22. (C)**

**Sol.** Multiply and divide by $x$ in the given expression, we get

$\lim_{x\to\infty}\frac{(1-\cos 2x)}{x^2}\frac{(3+\cos x)}{1}.\frac{x}{\tan 4x}$

$= \lim_{x\to 0}\frac{2\sin^2}{x^2}.\frac{3+\cos x}{1}.\frac{x}{\tan 4x}$

$= 2\lim_{x\to 0}\frac{\sin^2 x}{x^2}.\lim_{x\to 0}3+\cos x\cdot\lim_{x\to 0}\frac{x}{\tan 4x}$

$= 2.4\frac{1}{4}\lim_{x\to 0}\frac{4x}{\tan 4x} = 2.4.\frac{1}{4} = 2$

**23. (C)**

**Sol.** Intersection point of $2x-3y+4=0$ and $x-2y+3=0$ is

Since , P is the fixed point for given family of lines

So, PB = PA

$(\alpha-1)^2 + (\beta-2)^2 = (2-1)^2 + (3-2)^2$

$(\alpha-1)^2 + (\beta-2)^2 = 1+1 = 2$

$(x-1)^2 + (y-2)^2 = (\sqrt{2})^2$

$(x-a)^2 + (y-b)^2 = r^2$

**24. (C)**

**Sol.** $x^2 + y^2 - 4x - 6y - 112 = 0$ ...(1)

Centre, $c_1 = (2,3)$

Radius, $r_2 = 8$ units

$C_1C_2 = \sqrt{(2+3)^2 + (3+9)^2} = 13$ units

$r_1 + r_2 = 5 + 8 = 13$

$\therefore$ $C_1C_2 = r_1 + r_2$

Therefore there are three common tangents.

**25. (D)**

**Sol.** The end point of latus rectum of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is first quadrant is $\left(ae, \frac{b^2}{a}\right)$ and the tangent at this point intersects x-axis at $\left(\frac{a}{e}, 0\right)$ and y-axis at (0, a).

The given ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

Then $a^2 = 9, b^2 = 5$

$\Rightarrow$ $e = \sqrt{1-\frac{5}{9}} = \frac{2}{3}$

$\therefore$ and point of latus rectum is first quadrent is L(2, 5/3)

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$

It meets x-axis at A (9/2, 0) and y-axis at B (0,3)

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\times\frac{9}{2}\times 3 = \frac{27}{4}$

By symmetry area of quadrilateteral

$=$ $4\times(\text{Area}\,\Delta\,\text{OAB}) = 4\times\frac{27}{4} = 27$ sq. uints.

**26. (D)**

**Sol.** Let P(h, K) divides

OQ in the ration 1 : 3

Let any point Q on $x^2 = 8y$ is $(4t, 2t^2)$.

Then by section formula

$\Rightarrow$ $k = \frac{t^2}{2}$ and h = t

$\Rightarrow$ 2k = h$^2$

Required locus of P is $x^2 = 2y$

**27. (B)**

**Sol.** First 50 even natural number are

2, 4, 6…..,100

Variance = $\frac{\sum x_i^2}{N} - (\overline{x})^2$

$\Rightarrow$ $\sigma^2 = \frac{2^2+4^2+...+100^2}{50} - \left(\frac{2+4+...+100}{50}\right)^2$

$= \frac{4(1^2+2^2+3^2+...+50^2)}{50} - (51)^2$

$= 4\left(\frac{50\times 51\times 101}{50\times 6}\right) - (51)^2 = 3434 - 2601$

$\Rightarrow$ $\sigma^2 = 833$

**28. (A)**

**Sol.**

| *p* | *q* | *~q* | *p ↔ ~q* | *~(p ↔ ~q)* |
|---|---|---|---|---|
| *F* | *F* | *T* | *F* | *T* |
| *F* | *T* | *F* | *T* | *F* |
| *T* | *F* | *T* | *T* | *F* |
| *T* | *T* | *F* | *F* | *T* |

Clearly equivalent to P ↔ q

**29. (D)**

**Sol.** We know minimum value $|Z_2 + Z_2|$ is

$||Z_1| - |Z_2||$

Thus minimum value of $\left|Z + \frac{1}{2}\right|$ is

$\left||Z| - \frac{1}{2}\right| \le \left|Z + \frac{1}{2}\right| \le |Z| + \frac{1}{2}$

Since, $|Z| \ge 2$ therefore

$2 - \frac{1}{2} > \left|Z + \frac{1}{2}\right| < 2 + \frac{1}{2}$

$\Rightarrow$ $\frac{3}{2} < \left|z + \frac{1}{2}\right| < \frac{5}{2}$

**30. (D)**

**Sol.** Equation of circle $C \equiv (x-1)^2 + (y-1)^2 \equiv 1$

Radius of T = |y|

*T* touches C externally

Therefore,

Distance between the centres = sum of their radii

$\Rightarrow$ $\sqrt{(0-1)^2 + (y-1)^2} = 1 + |y|$

$\Rightarrow$ $(0-1)^2 + (y-1)^2 = (1 + |y|)^2$

$\Rightarrow$ $1 + y^2 + 1 - 2y = 1 + y^2 + 2|y|$

$2|y| = 1 - 2y$

If y > 0 then $2y = 1 - 2y \Rightarrow y = \frac{1}{4}$

If y > 0 then $-2y = 1 - 2y \Rightarrow y = 1$

(not Possible)

$\therefore$ $y = \frac{1}{4}$

## Physics

**31.(B)**

**Sol.** Percentage error in volume is

$$\frac{0.01}{15.12}\times100+\frac{0.01}{10.15}\times100+\frac{0.01}{5.28}\times100=0.35\%$$

**32.(D)**

**Sol.** $f=Cm^xk^y$. Writing dimensions on both sides,

$$[M^0L^0T^{-1}]=M^x[ML^0T^{-2}]^y=[M^{x+y}T^{-2y}]$$

Comparing dimensions on both sides, we have

$0=x+y$ and $-1=-2y$ $\Rightarrow$ $y=\frac{1}{2}, x=-\frac{1}{2}$

**Aliter:** Remembering that the frequency of oscillation of loaded spring is

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}(k)^{1/2}m^{-1/2}$$

which gives $x=-\frac{1}{2}$ and $y=\frac{1}{2}$

**33.(A)**

**Sol.** Let the particle be thrown up with initial velocity $u$.

Displacement ($s$) at any time $t$ is

$$S=ut-\frac{1}{2}gt^2$$

The graph should be parabolic downwards as shown in option (b).

**34.(B)**

**Sol.** Suppose $v$ be the velocity attained by the body after time $t_1$.

Then $v = u - gt_1$ (i)

Let the body reach the same point at time $t_2$. Now velocity will be downwards with magnitude $v$. Then

$-v = u - gt_2$ (ii)

(i) – (ii) $\Rightarrow$ $2v = g(t_2 - t_1)$

or $t_2-t_1=\frac{2v}{g}=\frac{2}{g}(u-gt_1)=2\left(\frac{u}{g}-t_1\right)$

**35.(B)**

**Sol.** Speed of train = $108\times\frac{5}{18}=30\text{ ms}^{-1}$

Let $\vec{V}_R$ and $\vec{V}_T$ represent the respective velocities of rain and train

Now, the relative velocity of rain w.r.t. person (train) is given by

$c_{R,T}=v_R-v_T \Rightarrow v_R+(-v_T)$

Let $\overrightarrow{OR}$ and $\overrightarrow{RT}$ represent the vector respectively magnitude and direction.

$OT^2 = OR^2 + RT^2 + 2\ OR \cdot RT \cos 120°$

$$=20^2+30^2-2\times20\times30\times\frac{1}{2}$$

$$400+900-600=700=\sqrt{700}\text{ ms}^{-1}$$

$$=10\sqrt{7}\text{ ms}^{-1}$$

**36.(A)**

**Sol.** For the person to be able to catch the ball, the horizontal component of velocity of the ball should be same as the speed of the person, i.e.,

$$v_0 \cos\theta = \frac{v_0}{2} \text{ or } \cos\theta \frac{1}{2} \text{ or } \theta = 60^o$$

**37.(D)**

**Sol.** Extension in the spring is

$$x = AB - R = 2R\cos 30^o - R = (\sqrt{3}-1)R$$

$$\text{Spring force}: F = kx = \frac{(\sqrt{3}+1)mg}{R} \times (\sqrt{3}-1)R$$
$$= 2mg$$

From the Figure, we have

$$N = (F + mg)\cos 30^o = \frac{3\sqrt{3}\,mg}{2}$$

**38.(C)**

**Sol.** Suppose a is the acceleration of block w.r.t. wedge.

For block :

$N + mA \sin 45° = mg \cos 45°$
(i)

For wedge : $N \sin 45° = mA$
(ii)

From (i) and (ii) A = 1 m/s$^2$

**39.(C)**

**Sol.** The driving force on the block

$$F_1 - F_2 = 10 - 2 = 8 \text{ N}$$

As the block is at rest the friction will be static and towards left.

$$f = f_s = 8 \text{ N}$$

If $F_1$ is removed only $F_2$ = 2 N is acting on the block.

If the block is at rest $f = F_2$ = 2 N this value of friction is within static range as in previous case even $f$ = 8 N is case.

**40.(A)**

**Sol.** If block is sliding with constant velocity

$$mg \sin\theta = \mu N + ma \sin\theta$$

and $N + ma \cos\theta = mg \cos\theta$

From (i), $N = m(g - a)\cos\theta$

$mg \sin\theta = \mu m(g - a)\cos\theta + ma \sin\theta$

$$10 \times \frac{1}{2} = \mu(10-2)\frac{\sqrt{3}}{2} + 2 \times \frac{1}{2} \Rightarrow \mu = \frac{1}{\sqrt{3}}$$

**41.(C)**

**Sol.** $P = Fv = M\dfrac{dv}{dt}v$

Hence, $v\,dv = \dfrac{P}{m}dt$

On integration, we find v ∝ $\sqrt{t}$

**42.(A)**

**Sol.** KE of blocks at B = PE at A – PE at B

$$\frac{1}{2}mv^2 = mgh - mg(h - 2r)$$

$v^2 = 2g(h - 2r)$
(i)

Also, $\frac{mv^2}{r} = x\,mg + mg$

or $v^2 = (x + 1)rg$

(ii)

Equating Eqs (i) and (ii), we get $2g(h - 2r) = (x + 1)gr$

or $2gh = (x + 1)gr + 4gr = (x + 5)gr$

$$h = \left(\frac{x+5}{2}\right)r$$

**43.(C)**

**Sol.** $mv = mv_1 + nmv_2$

$v = v_1 + nv_2 = v_2\ v_1$

$$\Rightarrow \quad v_2 = \frac{2v}{(n+1)},\ v_1 = \left(\frac{1-n}{1+n}\right)v$$

$$\frac{KE_1}{KE_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \left(\frac{n-1}{n+1}\right)^2$$

**44.(A)**

**Sol.** Applying the law of conservation of momentum,

$m_1v_1 = (m_1 + m_2)V$

(i)

where $v_1 = \sqrt{2gd}$ is the velocity with which $m_1$ collides with $m_2$. Therefore,

$$V = \frac{m_1}{(m_1+m_2)}\sqrt{2gd}$$

Now, let the centre of mass rise through a height $h$ after collision.

In this case, the kinetic energy of $m_1 + m_2$ system is converted into potential energy at maximum height $h$.

$$\Rightarrow \frac{1}{2}(m_1+m_2)V^2 = (m_1+m_2)gh$$

$$\Rightarrow \frac{1}{2}(m_1+m_2)\left\{\frac{m_1}{m_1+m_2}\right\}^2 2gd = (m_1+m_2)gh$$

$$\Rightarrow h = d\left\{\frac{m_1}{m_1+m_2}\right\}^2$$

**45.(D)**

**Sol.** $I = I_1 + I_2 + I_3$

$$I_1 = I_2 = \frac{3}{2}mr^2$$

$$I_1 = I_2 = \frac{3}{2}mr^2$$

$$I = I_1 + I_2 + I_3 = \frac{7}{2}mr^2$$

**46.(D)**

**Sol.** Given $\alpha_A = 2\alpha = 5\,\text{m/s}^2 \Rightarrow \alpha = \frac{5}{2}\,\text{m/s}^2$

Hence, acceleration of $B$, $a_B$ = $1(\alpha)\ = \frac{5}{2}\,\text{m/s}^2$

**47.(D)**

**Sol.** $m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = 2$

$$r_1 = 4r_2 \Rightarrow \frac{r_1}{r_2} = 4$$

$$T_A^2 \propto r_1^2 \text{ and } T_B^2 \propto r_2^3$$

$$\therefore \quad \frac{T_A}{T_B} = \left(\frac{r_1}{r_2}\right)^{3/2} = (4^3)^{1/2}$$

$$\Rightarrow \quad \frac{T_A}{T_B} = 8$$

**48.(B)**

**Sol.** $mg = mR\omega_2$

R = radius of earth

$$\omega = \sqrt{\frac{g}{R}}$$

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{64000}$$

$$= 2\pi\times 800\,\text{s} = \frac{2\pi\times 800}{3600}\text{h} = 1.36 = 0.14\,\text{h}$$

**49.(D)**

**Sol.** Work done $=\frac{1}{2}\text{stress}\times\text{strain}\times\text{volume}$

$$\frac{1}{2}\frac{F}{A}\frac{L}{L}(AL)=\frac{1}{2}Fl$$

**50.(C)**

**Sol.** $Y=\frac{Fl}{\pi r^2\Delta l}$ or $\Delta l=\frac{Fl}{\pi r^2 Y}$

$$\Delta l=\frac{1}{r^2},\Delta l'\propto\frac{2l}{(\sqrt{2}\,r)^2} \text{ or } \Delta l'\propto\frac{l}{r^2}$$

$$\therefore \quad \frac{\Delta l}{\Delta l'}=1$$

**51.(A)**

**Sol.** Let $\rho$ be the density of the liquid. Then

$F_1 = (\Delta P)\,A = \rho gh\,A$ (i)

In the second case, $F_2$ = rate of change of momentum

$\rho Au^2=\rho A(\sqrt{2gh})^2=2\rho ghA$ (ii)

$$\therefore \quad \frac{F_1}{F_2}=\frac{1}{2}$$

**52.(A)**

**Sol.** $W$ = weight of liquid.

$f_B$ = buoyant force on the ball

$mg$ = weight of the ball

$N$ = normal reaction between the ball and the surface

The free-body diagrams of the balls in each vessel are as follows.

At base, reaction force of buoyant force will act in downward direction

The forces acting at the base of each tank are

$F_A = W + f_B = W + mg$

$F_B = W + f_B = W + mg$

$F_C = W + f_B + N = W + mg$

Thus, $F_A = F_B = F_C$

**53.(C)**

**Sol.** According to Newton's law of cooling the rate of cooling depends the surrounding. It means that when the difference of temperature between the body and the surrounding is small, time required for same fall in temperature is more in comparison with the same fall at higher temperature difference between the body and surrounding. So according to problem $T_1 < T_2 > T_3$

**54.(C)**

**Sol.** Rate of flow of heat along $PQ$

$$\left(\frac{dQ}{dt}\right)_{PQ}=\frac{K_3A\Delta\theta}{l} \qquad \text{(i)}$$

Rate of flow of heat along $PRQ$

$$\left(\frac{dQ}{dt}\right)_{PRQ}=\frac{K_3A\Delta\theta}{2l}$$

Effective conductivity for series combination of two rods of same length

$$K_s=\frac{2K_1K_2}{K_1+K_2}$$

So $\left(\frac{dQ}{dt}\right)_{PRQ} = \frac{2K_1K_2}{K_1+K_2} \cdot \frac{A\Delta\theta}{2l}$

$$= \frac{K_1K_2}{K_1+K_2} \cdot \frac{A\Delta\theta}{l} \quad \text{(ii)}$$

Equating Eqs. (i) and (ii) $K_3 = \frac{K_1K_2}{K_1+K_2}$

**55.(B)**

**Sol.** For an adiabatic process $TV^{\gamma-1}$ = constant. Therefore,

$$\frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{\gamma-1}$$

$$\Rightarrow \quad T_2 = T_1\left[\frac{V_1}{V_2}\right]^{\gamma-1}$$

$$= 300\left[\frac{27}{8}\right]^{5/3\ \ 1} = 300\left[\frac{27}{8}\right]^{2/3} = 675\ \text{K}$$

$$\Rightarrow \quad \Delta T = 675 - 300 = 375\ \text{K}$$

**56.(A)**

**Sol.** Area enclosed by curve 1 < Area enclosed by curve 2 < Area enclosed by curve 3

∴ $Q_1 < Q_2 < Q_3$ (As $\Delta U$ is same for all the curves)

**57.(D)**

**Sol** $x = a \sin \omega t$

$$x = a\sin\frac{2\pi}{T}\times\frac{T}{8}$$

or $$x = a\sin\frac{\pi}{4} = \frac{a}{\sqrt{2}}$$

**58.(C)**

**Sol** $F = -\frac{dU}{dx} = -8\sin 2x$

For small oscillations, sin $2x$ = $2x$

i.e., $a = -16x$

Since $a\mu - x$, the oscillations are simple harmonic in nature

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{1}{16}} = \frac{\pi}{2}\text{s}$$

**59.(B)**

**Sol.** $f_0 - f_c = 2$

or $\frac{v}{2l} - \frac{v}{4l} = 2$ or $\frac{v}{4l} = 2$

or $\frac{v}{l} = 8$

When length of OOP is halved and that of COP is doubled, the beat frequency will be

$$f_o' - f_c' = \frac{v}{l} - \frac{v}{8l} = \frac{7}{8}\frac{v}{l} = \frac{7}{8}\times 8 = 7$$

**60.(B)**

**Sol.** Maximum particle velocity = 4 wave velocity

$A\omega = 4 f\lambda$

$y_0 2\pi f = 4 f\lambda$

$$\lambda = \frac{\pi y_0}{2}$$

## Chemistry

**61.(B)**

**Sol.**

| Element | % | No. of moles | Simple ratio | Whole No. |
|---|---|---|---|---|
| C | 82.8 | $\frac{82.8}{12}=6.9$ | $\frac{6.9}{6.9}=1$ | 2 |
| H | 17.2 | $\frac{17.2}{1}=17.2$ | $\frac{17.2}{6.9}=2.5$ | 5 |

$C_2H_5$

**62.(D)**

**Sol.** Moles of $Ca=\frac{30}{40}=0.75$ of which is more than any other.

**63.(D)**

**Sol.** (Graham's law)

**64.(A)**

**Sol.** $c=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3\times8.314\times300}{32\times10^{-3}}}=4.836\times10^{2}\,m\,s^{-1}$

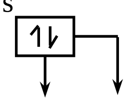
**65.(C)**

**66.(C)**

**67.(C)**

**Sol.** $He \longrightarrow 1s^2$

$1\,s^2$



| | | |
|---|---|---|
| $n$ | 1 | 1 |
| $l$ | 0 | 0 |
| $m$ | 0 | 0 |
| $s$ | $+\frac{1}{2}$ | $-\frac{1}{2}$ |

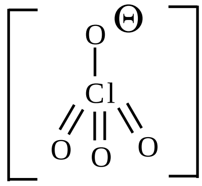
**68.(A)**

**69.(D)**

**Sol.** $N_2O_5 \equiv [NO_2]^+\,[NO_3]^-$

**70.(D)**

**Sol.**




**71.(B)**

**72.(B)**

**73.(C)**

**74.(A)**

**Sol.** Aim Reaction :-

$2C+H_2 \rightarrow C_2H_2$

Adding (i) and (ii)

$2C+H_2+2SO_2 \rightarrow 2CO_2+H_2O;$

$\Delta H=-1072.2\ kJ$

Reversing eq$^{\underline{n}}$ (iii)

$2CO_2+H_2O \rightarrow C_2H_2+2SO_2;$

$\Delta H=+1297\ kJ$

Adding

$2C+H_2 \rightarrow C_2H_2 \quad \Delta H=+224.8\ kJ$

**75.(D)**

**Sol.** pH of HCl = 2

$[H^+]$ = $10^{-2}$ M

pH of NaOH = 12

$[OH^-]$ = $10^{-2}$ M

After Reaction

$HCl+NaOH \rightleftharpoons NaCl+H_2O$

100 × 0·01

=1 eq

$[OH^-]$ after reaction $=\frac{1}{500}=2\times10^{-3}M$

pOH = $-\log[OH^-]$ = $-\log[2\times10^{-3}]$ = 2.7

pH = 14 – 2.7

= 11.3

**76.(D)**

**Sol.** $HA \rightleftharpoons H^+ + A^-$

$C \quad O \quad O$

$C(1-\alpha) \quad C\alpha \quad C\alpha$

$$k = \frac{C\alpha \cdot C\alpha}{C(1-\alpha)} = C\alpha^2$$

$$C = 10^{-2} M$$

$$\alpha = \frac{0.001}{100} = 10^{-5}$$

$$k = 10^{-2} \times (10^{-5})^2 = 10^{-12}$$

**77.(C)**

**78.(A)**

**79.(D)**

**80.(B)**

**Sol.** Due to Single Bond b/w two oxygen atom $H_2O_2$ has higher Bond length.

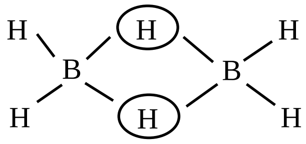
**81.(A)**

**Sol.** $Ca(OCl)_2$

**82.(D)**

**83.(B)**

**Sol.**




**84.(A)**

**Sol.** Due to Resonance.

**85.(B)**

**Sol.** Due to the presence of electron withdrawing group

**86.(A)**

**87.(A)**

**Sol.** $CH_3-CH_2-Cl \rightarrow CH_3-\dot{C}H_2 + \dot{C}l$

**88.(B)**

**Sol.** $C_6H_5CH_3 \rightarrow C_6H_3COOH \rightarrow$

$C_6H_5COONa \rightarrow C_6H_6$

**89.(D)**

**90.(A)**

**Sol.** Other does't follow (un+2)ITe's rule