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ENGINEERING & MEDICAL COACHING

Test Solution

Booster Engineering

Test Code : PT01-1617-BE

Answer & Solution Mathematics

1. B. $\theta \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan\theta < 1$ **and** $\cos\theta > 1$.

Let $\tan\theta = 1 - x$ and $\cot\theta = 1 + y$, where $x, y > 0$ and are very small, then $t_1 = (1-x)^{1-x}, t_2 = (1-x)^{1-y}, t_3 = (1+y)^{1-x}, t_4 = (1+y)^{1+y}$

Clearly, $t_4 > t_3$ and $t_1 > t_2$. Also $t_3 > t_1$.

Thus, $t_4 > t_3 > t_1 > t_2$

2. D. From the given relation, $m + n = 2\tan\theta,\ m - n = 2\sin\theta$

$\Rightarrow m^2 - n^2 = 4\tan\theta\sin\theta$ ....(1)

Also

$4\sqrt{mn} = \sqrt{\tan^2\theta - \sin^2\theta} = 4\sin\theta\tan\theta$ ....(2)

From Eqs. (1) and (2), we get $m^2 - n^2 = 4\sqrt{mn}$

3. C. The given equation is $\tan x + \sec x = 2\cos x$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2 - 2\sin^2 x$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{1}{2}, -1$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \in [0, 2\pi]$

But for $x = \frac{3\pi}{2}$ the given equation is not satisfied, as then $\infty - \infty$ is an indeterminate from.

Therefore, there are only two solution.

4. A. Given that 4A + A + A =180^o $\Rightarrow$ A = 30^o

$\Rightarrow$ Angles are 120^o, 30^o, 30^o

$\Rightarrow \frac{\sin 120^o}{a} = \frac{\sin 30^o}{b} = \frac{\sin 30^o}{c} = 2R$ (say)

$\Rightarrow \frac{a}{a+b+c} = \frac{\sin 120^o}{\sin 120^o + \sin 30^o + \sin 30^o}$

$= \frac{\sqrt{3}}{2+\sqrt{3}}$

5. A. $|x|^2 - 3|x| + 2 = 0$

$\Rightarrow (|x| - 2)(|x| - 1) = 0$

$\Rightarrow |x| = 1$ *or* 2

$\Rightarrow x = \pm 1, \pm 2$

6. B. Given that $|z_1|$=12. Therefore, z_1 lies on a circle with centre (0, 0) and radius 12 units. As $|z_2 - 3 - 4i| = 5$ so z_2 lies on a circle with centre (3, 4) and radius 5 units

Then $|z_1 - z_2| = AB = OA - OB =$ $12 - 2(5) = 2$. As it is the minimum value, we must have $|z_1 - z_2| \geq 2$.

7. B. $\left(\frac{x-1}{-2}\right)^3 = 1$

$\Rightarrow \frac{x-1}{-2} = 1, \omega, \omega^2$

$\Rightarrow x = -1, 1 - 2\omega,\ 1 - 2\omega^2$

8. B. If x, y and z are in G.P. $(x, y, z > 1)$, then $\log x,\ \log y,\ \log z$ are in A. P. Hence.

$1 + \log x, 1 + \log y, 1 + \log z$ will also be in A. P.

$\Rightarrow \frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ will be in H.P.

9. B **(Step - 1)**

$2\log_{10} x - \log_x 0.01 = \log_{10} x - \frac{\log_{10} 0.01}{\log_{10} x}$

$= 2\log_{10} x + \frac{2}{\log_{10} x}$

$= 2\left[\log_{10} x + \frac{1}{\log_{10} x}\right]$

[Hence $x > 1 \Rightarrow \log_{10} x > 0$**]**

(Step - 2) Now,

A.M. $\geq$ G.M.

$\Rightarrow \frac{\log_{10} x + \frac{1}{\log_{10} x}}{2} \geq \left(\log_{10} x \frac{1}{\log_{10} x}\right)^{1/2}$

$\Rightarrow \log_{10} x + \frac{1}{\log_{10} x} \geq 2$

10. C. The letter of word COCHIN in alphabetic order are C, C, H, I, N, O. Fixing first letter C and keeping C at second plane, rest 4 can be arranged in 4! Ways.

Similarly, the total number of words staring with CH, Cl. CN is 4! In each case.

Then fixing first two letters as Co, next four places when filled in alphabetic order gives the word COCHIN.

Therefore, number of words coming before COCHIN is $4! \times 4 = 4 \times 24 = 96$.

11. C. The given expression is $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$. We know that

$$(x+a)^n+(x-a)^n = 2\,[{}^nC_0x^n+{}^nC_2x^{n-2}a^2+{}^nC_4x^{n-4}a^4+...]$$

Therefore the given expression is equal to

$$2[{}^5C_0x^5+{}^5C_2x^3(x^3-1)+{}^5C_4x(x^3-1)^2].$$

Maximum power of x involved here is 7, also only +ve integral power of x are involved, therefore the given expression is a polynomial of degree 7.

12. D. The maximum of two numbers will be less than 4 or at least one of the numbers is less than 4.

$\therefore$ P(at least one numbers < 4) = 1– P (both the numbers ≥ 4)

$$=1-\frac{3}{6}\times\frac{2}{5}$$

$$=1-\frac{6}{30}$$

$$==1-\frac{1}{5}=\frac{4}{5}$$

13. C. Given line L : $2x + y = k$ passes through point (say P) which divides a lone segment (say AB) in ratio 3 : 2, where A (1, 1) and B (2, 4).

Using section formula, the coordinate of the point O which divides AB internally in the ratio 3 : 2 are

$$P\left(\frac{3\times2+2\times1}{3+2},\frac{3\times4+2\times1}{3+2}\right)=P\left(\frac{8}{5},\frac{14}{5}\right).$$

Also, since the line L passes through P, hence substituting the coordinates of $P\left(\frac{8}{5},\frac{14}{5}\right)$ in the equation of $L: 2x+y=k,$ we get

$$2\left(\frac{8}{5}\right)+\left(\frac{14}{5}\right)=k$$

$$\therefore \quad k=6$$

14. B. Take any point B (0, 1) on given line

Equation of AB'

$$y-0=\frac{-1-0}{0-\sqrt{3}}(x-\sqrt{3})$$

$$\Rightarrow \quad -\sqrt{3}y=-x+\sqrt{3}\Rightarrow x-\sqrt{3}y=\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}y=x-\sqrt{3}$$

Alternate Solution

Slope of given line is $-\frac{1}{\sqrt{3}}$.

From figure, equation of reflected ray is

$$y=\frac{1}{\sqrt{3}}(x-\sqrt{3})\Rightarrow\sqrt{3}y=x-\sqrt{3}$$

15. A. **I** $x-3-2\sqrt{x-4}\geq 0$

$x-3\geq 2\sqrt{x-4}$

$(x-3)^2\geq 4(x-4)$

$x^2-6x+9\geq 4x-16$

$x^2-16x+25\geq 0$

$(x-5)^2\geq 0$

$x\in R$ but x>4

$\Rightarrow [4,\infty)$ … (i)

II $(x-3)+2\sqrt{x-4}\geq 0$

$(x-3)^2\geq h(x-4)$

$x^2-6x+9-4x+16\geq 0$

$x^2-1x+25\geq 0$

$(x-5)^2\geq 0$

$x\in R$ but as $x>4$

$\Rightarrow [4,\infty)$ …(ii)

Hence, I & II

$[4,\infty)$

16. C. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{4\sin^4 x}$

$$=\lim_{x\to 0}\frac{x}{4\sin^4 x}\left[\frac{2\tan x}{1-\tan^2 x}-2\tan x\right]$$

$$=\lim_{x\to 0}\frac{x\tan^3}{2\sin^4 x\left(1-\tan^2 x\right)}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{x}{\sin x}\frac{1}{\cos^3 x}\frac{1}{1-\tan^2 x}$$

$$=\frac{1}{2}1\times\frac{1}{1^3}\times\frac{1}{1-0}=\frac{1}{2}.$$

17. C. Figure

Let equation of hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Where, $2ae = 4$ and $e = 2$

$\Rightarrow \quad a = 1$

$\because \quad a^2e^2 = a^2 + b^2$

$\Rightarrow \quad 4 = 1 + b^2$

$\therefore \quad b^2 = 3$

Thus, equation of hyperbola is $\frac{x^2}{1}-\frac{y^2}{3}=1$

or $3x^2 - y^2 = 3$

18. A. $y=(\sin x)^{\tan x}$

Taking log and differentiating with respect to x

$$\Rightarrow \frac{dy}{dx}=(\sin x)^{\tan x}[1+\sec^2 x\log\sin x]$$

19. D. Distance between their centre

$$k+1=\sqrt{1+(k+1)^2}$$

$$\Rightarrow \quad k+1=\sqrt{1+k^2+1-2k}$$

$$\Rightarrow \quad k+1=\sqrt{k^2+2-2k}$$

$$\Rightarrow \quad k^2+1+2k=k^2+2-2k$$

$$\Rightarrow \quad k=\frac{1}{4}$$

So, the radius or circle T is k i.e., $\frac{1}{4}$

20. A. For parabola, $y^2 = 4x$

Let $y=mx+\frac{1}{m}$ is tangent line and it touches the parabola $x^2=-32y$.

$$\therefore \quad x^2=-32\left(mx+\frac{1}{m}\right)$$

$$\Rightarrow \quad x^2+32mx+\frac{32}{m}=0$$

$$\because \quad D=0$$

$$\therefore \quad (32\,\mathrm{m})^2-4.\left(\frac{32}{m}\right)=0$$

$$\Rightarrow \quad m^3=\frac{1}{8}$$

$$\Rightarrow \quad m=\frac{1}{2}$$

21. C. $5|x|-x^2-6\geq 0$

$x^2-5|x|+6\leq 0$

Case I $x>0$

$x^2-5x+6\leq 0$

$(x-2)(x-3)\leq 0$

$2\leq x\leq 3$

Case II $x>0$

$x^2+5x+6\leq 0$

$(x+2)(x+2)\leq 0$

$-3\leq x\leq -2$

22. D. Given, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Passes through P(–3,1) and $e=\sqrt{\frac{2}{5}}$

$$\therefore \quad \frac{9}{a^2}+\frac{1}{b^2}=1 \text{ and } e^2=1-\frac{b^2}{a^2}$$

$$\Rightarrow \quad \frac{9}{a^2}+\frac{5}{3a^2}=1 \text{ and } \frac{2}{5}=1-\frac{b^2}{a^2}$$

$$\Rightarrow \quad \frac{27+5}{3a^2}=1 \text{ and } \frac{b^2}{a^2}=\frac{3}{5}$$

$$\Rightarrow \quad a^2=\frac{32}{5} \text{ and } b^2=\frac{32}{5}$$

$\therefore$ Equation on ellipse

$$\Rightarrow \quad \frac{3x^2}{32}+\frac{5y^2}{32}=1$$

$$\Rightarrow \quad 3x^2+5y^2=32$$

23. C. Given

(i) A line through (1, 2) meets the coordinate axes at P and Q.

(ii) The area of Δ OPQ is minimum.

To find the slope of line PQ.

Let m be the slope of the line PQ. Then the equation of PQ is

$$y-2=m(x-1)$$

Now, PQ meets X-axis at $P\left(1-\frac{2}{m},0\right)$ and Y-axis as Q(0, 2 – m).

$$\Rightarrow\ OP=1-\frac{2}{m}\ \text{and}\ OQ=2-m$$

Also, are of Δ OPQ = $\frac{1}{2}$(OP)(OQ)

$$=\frac{1}{2}\left|\left(1-\frac{2}{m}\right)(2-m)\right|$$

$$=\frac{1}{2}\left|2-m-\frac{4}{m}+2\right|$$

$$=\frac{1}{2}\left|4-\left(m+\frac{4}{m}\right)\right|$$

Let $f(\text{m})=4-\left(m+\frac{4}{m}\right)$

$$\Rightarrow\quad f'(\text{m})=-1+\frac{4}{m^{32}}$$

Now, $f'(\text{m})=0$

$$\Rightarrow\quad m^2=4\Rightarrow m=\pm 2$$

$$\Rightarrow\quad f(2)=0\ \text{and}\ f(-2)=8$$

Since , the area cannot be zero, hence the required value of m is –2.

24. D.

$X=\{4^n-3\text{n}-1:\text{n}\in\text{N}\}$

$X=\{0,9,54,243,....\}$ [put n = 1,2,3....]

$\text{Y}=\{9(\text{n}-1):\text{n}\in\text{N}\}$

$\text{Y}=\{0,1,18,27,....\}$

It is clear that $X\subset Y$.

$\therefore\quad X\cup Y=Y$

25. A. **To find** The shortest distance between y – x =1 and x = y^2 along the common normal.

$\therefore$ Tangent at P is parallel to

y = x + 1 .....(i)

$\therefore$ Slope of tangent at $P(\text{t}^2,\text{t})$.

$$\Rightarrow\ \frac{dy}{dx}=\left(\frac{1}{2y}\right)_{(\text{t}^2,\text{t})}=\frac{1}{2t}\quad .....(ii)$$

$$\Rightarrow\ \frac{1}{2t}=1\ \text{[Eqs. (i) and (ii) are parallel]}$$

$$\Rightarrow\ t=\frac{1}{2}$$

$$\therefore\ P\left(\frac{1}{4},\frac{1}{2}\right)$$

Shortest distance $=|\text{PQ}|=\dfrac{\left|\frac{1}{4}-\frac{1}{2}+1\right|}{\sqrt{1+1}}=\dfrac{3}{4\sqrt{2}}$

Hence, shortest distance is $\frac{3\sqrt{2}}{8}$.

Alternate Solution

Give, $x-y+1=0$ .....(i)

and $x=y^2$

$$\Rightarrow\quad 1=2y\frac{dy}{dx}$$

$$\Rightarrow\quad \frac{dy}{dx}=\frac{1}{2y}=\text{slope of the line (i)}$$

$$\Rightarrow\quad \frac{1}{2y}=1\Rightarrow y=\frac{1}{2}$$

$$\Rightarrow\quad x=\left(\frac{1}{2}\right)^2=\frac{1}{4}$$

$\Rightarrow$ The point is $(\text{x},\text{y})=\left(\frac{1}{4},\frac{1}{2}\right)$

$\therefore$ The shortest distance is

$$\frac{\left|\frac{1}{4}-\frac{1}{2}+1\right|}{\sqrt{1+1}}=\frac{3}{4\sqrt{2}}=\frac{3\sqrt{2}}{8}$$

26. B. Given graph is symmetrical about the line x = 2

$\therefore \quad f(2+x) = f(2-x)$

27. A. $\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}} \left(\frac{0}{0}\text{from}\right)$

$= \lim_{x\to\frac{\pi}{4}} \frac{f(\cos^2 x)2\sec x \sec x \tan x}{2x}$

$= \frac{2f(2)}{\pi/4} = \frac{8f(2)}{\pi}$

Since $f(x) = g(x)\sin x$

$f'(x) = g'(x)\sin x + g(x)\cos x$

and

$f''(x)g''(x)\sin x + 2g'(x)\cos x - g(x)\sin x \Rightarrow f''(0) = 0$

Thus

$\lim_{x\to 0}[g(x)\cos x - g(g)\operatorname{cosec} x] = 0\, f''(0)$

28. D. Sum of 16 observation $= 16\times 16 = 256$

Sum of resultant 18 observation = $256 - 16 + (3+4+5) = 252$

Mean of observation = $\frac{252}{18} = 14$

29. C. Statement I

Analysis given, a parabola $y^2 = 16\sqrt{3}x$ and an ellipse $2x^2 + y^2 = 4$.

To find The equation of common tangent to the given parabola and the ellipse. This can be very easily done by comparing the standard equation of tangents. Standard equation of tangent with slope '***m***' to the parabola $y^2 = 16\sqrt{3}x$ is

$y = mx + \frac{4\sqrt{3}}{m}$ … (i)

Standard equation of tangent with slope '***m***' to the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ is

$y = mx \pm \sqrt{2m^2 + 4}$ …(ii)

If a line L is a common tangent to both parabola and ellipse, then

L should be tangent to parabola, i.e., its equation should be like Eq. (i).

L Should be tangent to ellipse i.e., its equation should be like Eq. (ii).

i.e., L must be like both of the Eqs. (i) and (ii).

Hence, comparing Eqs. (i) and (ii), we get

$\frac{4\sqrt{3}}{m} = \pm\sqrt{2m^2 + 4}$

On squaring both sides we get

$\Rightarrow \quad m^2(2m^2 + 4) = 48$

$\Rightarrow \quad m^4 + 2m^2 + 24 = 0$

$\Rightarrow \quad (m^2 + 6)(m^2 - 4) = 0$

$\Rightarrow \quad m^2 = 4$

$[\because \; m^2 \neq -6]$

$\therefore \quad m = \pm 2$

Statement II

In Statement II, we have already seen that, if the line $y = mx + \frac{4\sqrt{3}}{m}$ is a common tangent to the parabola $y^2 = 16\sqrt{3}\times$ and the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$, then is satisfies the equation $m^4 + 2m^2 - 24 = 0$. Hence, Statement II is also correct but is ot able to explain the Statement I, it is an intermediate step in the final answer.

30. D. ~[~ s∨ (~ r^ s)]

=s^~(~r^S)

=s^(r∨~s)

=(s^r) ∨ (s^~s)

=(s^r) ∨ 0

=s^r

Answer & Solution Physics

31. (D) Momentum, $p = mv = MLT^{-1} = ML^{-3}L^4T^{-4}T^3$

$= DV^4F^{-3}$

32.(D) $X = M^{-1}L^3T^{-2}$

$$\frac{\Delta X}{X} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

$= 2 + 3 \times 3 + 2 \times 4 = 19$

33.(D) Distance covered by the object in first 2 s

$$h_1 = \frac{1}{2}gt^2 = \frac{1}{2}\times 10 \times 2^2 = 20\text{m}$$

Similarly, distance covered by the object in next 2 s will also be 20 m, hence the required height = H – 20 – 20

= H – 40 m

34.(C) Maximum acceleration will be from 30 to 40 s, because slope in this interval is maximum.

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{60 - 20}{40 - 30} = 4ms^{-2}$$

35.(A) $\vec{v}_c = 25\hat{i}, \vec{v}_{b/c} = 25\sqrt{3}\,\hat{j}$

$$\vec{V}_{b/c} = \vec{V}_b - \vec{V}_c \Rightarrow \vec{V}_b = \vec{V}_{b/c} + \vec{V}_c$$

$$\Rightarrow \quad \vec{v}_b = 25\hat{i} + 25\sqrt{3}\,\hat{j}$$

$$|v_b| = \sqrt{25^2 + (25\sqrt{3})^2} = 50\,\text{km h}^{-1}$$

$$\tan\theta = \frac{25}{25\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

36.(B) H = 100 m, R = 2 × 200 = 400 m

$$\tan\theta = \frac{4H}{R} \Rightarrow \tan\theta = \frac{4\times 100}{400} = 1$$

$$\Rightarrow \quad \theta = 45^\circ \quad \left[\because \frac{H}{R} = \frac{\tan\theta}{4}\right]$$

37.(A) Acceleration of the system :

$$a = \frac{P}{M+m} \qquad \text{(i)}$$

The FBD of mass m is shown in the figure.

$R \sin \beta = ma$ (ii)

$R \cos \beta = mg$ (iii)

From Eqs. (ii) and (iii), we get

$a = g \tan \beta$

Putting the value of a in (i), we get

$P = (M + m)g \tan \beta$

38.(B) Here

$$a = \frac{m_1 - m_2}{m_1 + m_2}g,$$

if $m_1 > m_2$.

But a = $g/8$

$$\frac{g}{8} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

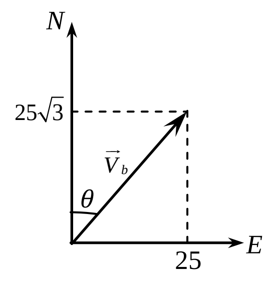
or $\quad m_1 : m_2 = 9 : 7$

39.(C) Given horizontal force F = 25 N and coefficient of friction between block and wall (μ) = 0.4.

We know that at equilibrium horizontal force provides the normal reaction to the block against the wall. Therefore, normal reaction to the block (R) = F = 25 N.

We also know that weight of the block (W) = Frictional force

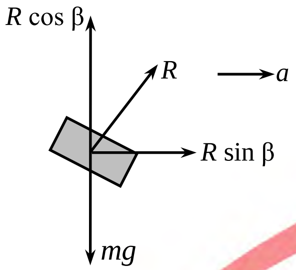
= μR = 0.4 × 25 = 10 N.



40.(D) Net force, $\vec{F} - mg\hat{i} = m(a_0\hat{j} - 2a_0\hat{i})$

$$\Rightarrow \vec{F} = m\left(-2a_0\hat{i} + (a_0 + g)\hat{j}\right) = m(-g\hat{i} + \frac{3g}{2}\hat{j})$$

$$\Rightarrow \quad F = m\sqrt{g^2 + \left(\frac{3g}{2}\right)^2} = \frac{\sqrt{13}mg}{2}$$



41.(C) Let us assume that the displacement of the body is directly proportional to t^n, i.e.,

$$s = Kt^n, v = \frac{ds}{dt} = Knt^{n-1}$$

and $$a = \frac{dv}{dt} = Kn(n-1)t^{n-2}$$

Force $F = ma = mKn(n-1)t^{n-2}$

Power, $P = Fv = [mKn(n-1)t^{n-2}]\ [Knt^{n-1}]$

$= mkn^2(n-1)t^{2n-3}$

As power is constant, i.e, independent of time, hence

$$2n - 3 = 0 \text{ or } n = \frac{3}{2} \text{ or } s \propto t^{\frac{3}{2}}$$

42.(D) $f(x) = -\frac{dU}{dx}(x)$ or $U(x) = -\int F(x)dx$

Here $F(x) = -kx$, where k is a positive constant.

43.(A) $m_1u_1 = m_2v_2$

$$\frac{1}{2}m_2v_2^2 = \frac{1}{2}\left[\frac{1}{2}m_1u_1^2\right]$$

$$\Rightarrow (m_2v_2)v_2 = \frac{1}{2}(m_1u_1)u_1 \Rightarrow v_2 = \frac{u_1}{2}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - 0}{u_1 - 0} = \frac{v_2}{u_1} \quad \Rightarrow \quad e = \frac{1}{2}$$

44.(A) In this problem, the velocity of the earth before and after the collision may be assumed zero. Hence, coefficient of restitution will be

$$e^n = \frac{v_1}{v_0} \times \frac{v_2}{v_1} \times \frac{v_3}{v_2} \times \ldots \times \frac{v_n}{v_n - 1}$$

where v_n is the velocity after nth rebounding and v_0 is the velocity with which the ball strikes the earth for the first time.

Hence,

$$e^n = \frac{v_n}{v_0} = \frac{\sqrt{2gh_n}}{\sqrt{2gh_0}}$$

where h_n is the height to which the ball rises after nth rebounding. Hence,

$$e^n = \frac{v_n}{v_0} = \frac{\sqrt{h_n}}{\sqrt{h_0}}$$

45.(A) Mass of each of the four parts $= \frac{M}{3}$

Mass of the plate including the cut piece $= \frac{4M}{3}$

Moment of inertia of the whole plate (including the cut piece) about the said axis $= \left(\frac{4M}{3}\right)\frac{l^2}{6}$

Now moment of inertia of the remaining portion should be ¾ of the above = $Ml^2/6$

46.(A) $T_1 - mg = ma$

$r(T_2 - T_2) = I\alpha$

$Mg - T_3 = Ma$

$r(T_3 - T_2) = I\alpha$

and $a = R\alpha$

From Eqs. (ii) and (iv), we get

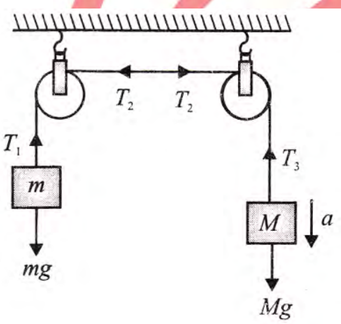
$$T_3 - T_1 = \frac{2Ia}{R^2}$$

From Eqs. (i) and (iii), we get

$$(M - m)g = (M + m)a + T_3 - T_1$$

$$(M - m)g = (M - m)a + \frac{2Ia}{r^2}$$

$$a = \frac{(M - m)g}{\left(M + m + \frac{2I}{r^2}\right)}$$



47.(A) Total mechanical energy is given by

$$E = K + u = -\frac{GMm}{2a} - \frac{GMm}{a} = \frac{-GMm}{2a}$$

$$\frac{GM}{a} = v^2 \Rightarrow E = -\frac{1}{2}mv^2$$

48.(C) $F_G = \frac{Gm^2}{4R^2} \Rightarrow \frac{mv^2}{R} = \frac{Gm^2}{4R^2}$

$$\therefore v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

49.(C) Work done $= \frac{1}{2}F \times \text{Extension}$

$$= \frac{1}{2} \times \frac{YA}{L} \times 1 \qquad Y = \frac{F \times L}{A \times 1}$$

$$= \frac{YA}{2L} \qquad F = \frac{YA}{L}$$

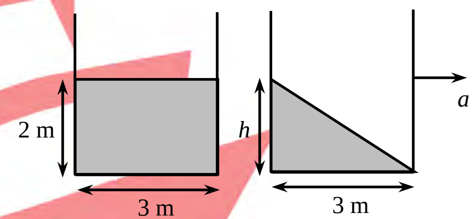
50.(C) $m = al\rho$, m and ρ are constants.

$$\therefore \frac{a_1}{a_2} = \frac{l_2}{l_1} = \frac{3}{2}$$

Now, $Y = \frac{Fl}{a\Delta l}$ or $\Delta l \propto \frac{l}{a}$

or $\frac{\Delta l_1}{\Delta l_2} = \frac{l_1}{l_2} \times \frac{a_2}{a_1} = \frac{2}{3} \times \frac{2}{3}$ or $\frac{\Delta l_1}{\Delta l_2} = \frac{4}{9}$

51.(B)



Volume equality gives

$$2 \times 3 = \frac{1}{2} \times h \times 3 \quad \Rightarrow h = 4 \text{ m}$$

$$\therefore \quad \tan\theta = \frac{4}{3} = \frac{a}{g} \quad \Rightarrow a = \frac{4}{3}g$$

52.(B) Using equation of continuity, we have $v_2 = \frac{A_1}{A_2}v_1$

From Bernoulli's theorem,

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$$

$$p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \Rightarrow g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2)$$

$$\Rightarrow \quad 60 = \left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2 \Rightarrow \frac{A_1}{A_2} = \frac{4}{1}$$

53.(B) If the sheet is heated then both d_1 and d_2 will increase since the thermal expansion of isotropic solid is similar to true photographic enlargement.

54.(D) According to Newton's law of cooling

$$\frac{\theta_1-\theta_2}{t} \propto \left[\frac{\theta_1+\theta_2}{2}-\theta\right]$$

For the first condition

$$\frac{80-60}{60} \propto \left[\frac{80+60}{2}-30\right] \qquad \text{(i)}$$

and for the second condition

$$\frac{60-50}{t} \propto \left[\frac{60+50}{2}-30\right] \qquad \text{(ii)}$$

By solving Eqs. (i) and (ii), we get t = 48 s.

55.(A) From ideal gas equation,

$$PV = \mu RT$$

$$\therefore \quad P = \frac{\mu R}{V} T$$

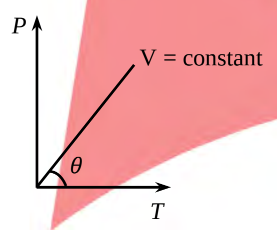
Comparing, this equation with $y = mx$

Slope of line, $\tan\theta = m = \mu R/V$

$$i.e., \quad V \propto \frac{1}{\tan\theta}$$

If means line of smaller slope represents greater volume of gas.

For the given problem figure



Points 1 and 2 are on the same line, so they will represents same volume, i.e., $V_1 = V_2$.

Similarly points 3 and 4 are on the same line, so they will represents same volume, i.e., $V_3 = V_4$.

But $V_1 > V_3$ $(= V_4)$ or $V_2 > V_3$ $(= V_4)$ as slope of line 1–2 is less than that of 3–4.

56.(C) In second part there is a vacuum, i.e., $P = 0$. So work done in expansion = $P\Delta V = 0$.

Also, $\Delta Q = 0$. From the first law of thermodynamics, $\Delta U = 0$

i.e., temperature of an ideal gas remains same due to free expansion.

57. (B)

58.(C) $v_1 = \omega\sqrt{a^2 - x_1^2}, \quad v_2 = \omega\sqrt{a^2 - x_2^2}$

We get, $a = \sqrt{\dfrac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$

59.(D) $y = 4\sin\left(4\pi t - \dfrac{\pi}{16}x\right)$

$$\omega = 4\pi, k = \pi/16$$

$$v = \frac{\omega}{k} = \frac{4\pi}{\pi/16} = 64 \text{ cm/s}$$

In positive x-direction

60.(C) Node means a point at which medium particles do not displace from its mean position and antinode mean a point at which particles oscillate with maximum possible amplitude. Nodes and antinodes are obtained for both types of stationary waves, transverse and longitudinal. Hence , options (A) and (B) both are wrong . To obtain a stationary wave, two waves travelling in opposite directions, having same amplitude, same frequency are required. They must have same nature, means either both of the wave should be longitudinal or both of them should be transverse. Hence, option (C) is correct.

Answer & Solution Chemistry

61. (B) $N^{3-} > O^{2-} > F^{\ominus}$

↓ Greater no. of electrons

62. $3s^1$ or $3s^2$

↓

$1s^2\ 2s^2\ 3s^1$ or $3s^2$

11 or 12

63. (D) Both have same no. of electrons

64. (C)

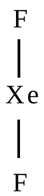
65. s > p > d > f

66. (A)

67. (B) Due to greater p character.

68. (A) NH_3 (lone pair) ⟶ Having alone pair.

69. (C)



O = C = O

Both are linear.

70. (C) at this $cond^n$ $\frac{an^2}{v^2}$ and nb is neglected.

$$\left(p+\frac{an^2}{v^2}\right)(v-rb)=nRT$$

$$\boxed{Pv = nRT}$$

71. (B)

72. (A)

73. (A) $\Delta E = Q + W$

ΔE → Internal energy change, Q → Heat, W → Work

74. (C)

75. $K_P = K_C(RT)^{\Delta n}$

$$\frac{K_P}{K_C}=(RT)^{1-2}$$

$$\boxed{\frac{K_P}{K_C}=\frac{1}{RT}}$$

76. $K_2=\frac{1}{K_1^2}$

as the 2^{nd} reaction is reversed and multiplied by 2 to get 2^{nd}.

77. $NH_2^{\ominus}+H^{\oplus}\longrightarrow NH_3$

($NH_2^{\ominus}$ ↓ Base)

78. $NaCN < NH_4Cl < NaCl < HCl$

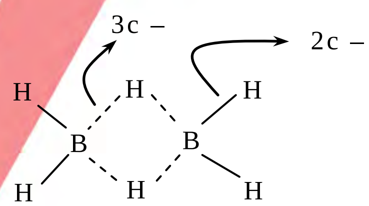
79. (D) NH_4OH or CH_3COOH is not a salt.

80. (C) Due to size difference.

81. $Na+NH_3\longrightarrow Na^{+}+(NH_3)e^{-}$

82. (B) Due to inert pair effect.

83.



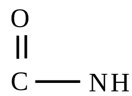
84. $K_2SO_4\cdot Al_2(SO_4)_3\cdot 2H\cdot H_2O$

↓ H_2

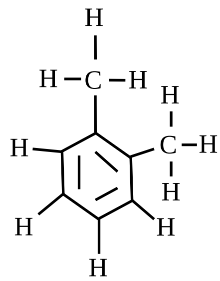
$2K^{+}+SO_4^{2-}+2Hl^{3+}$

85. $B(OH)_3+H_2O\longrightarrow H^{\oplus}+B(OH)_4^{\ominus}$

86.

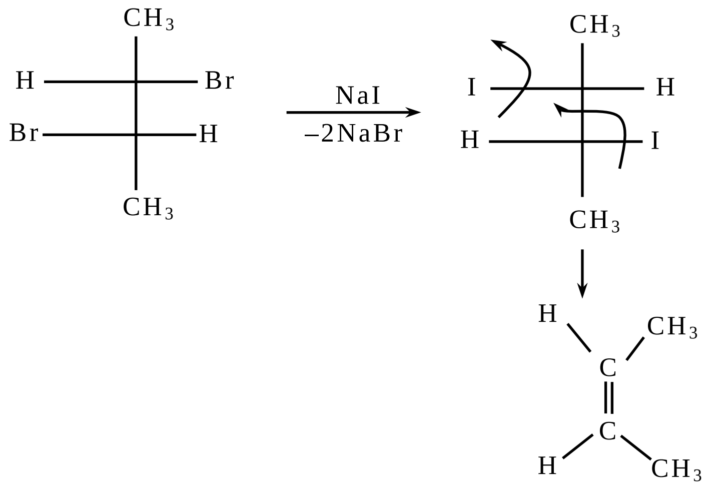


87.



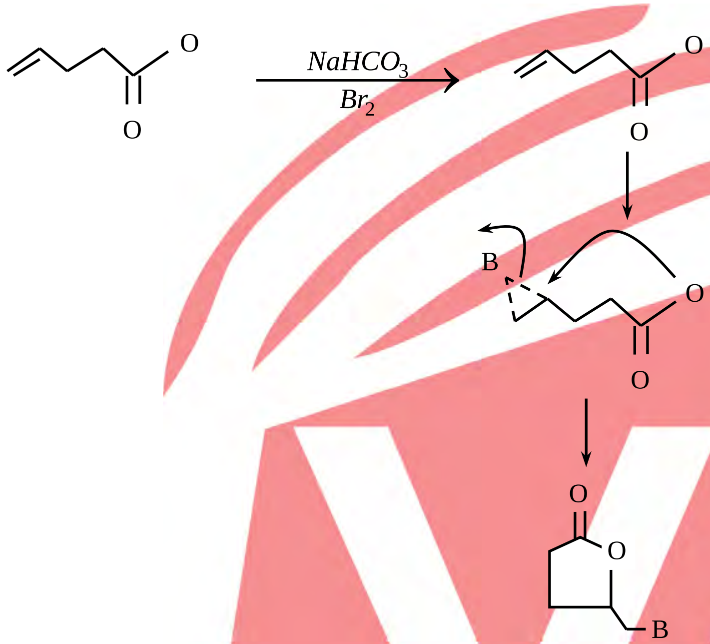
88. (C)

89.



CH3
H
Br
Br
H
CH3
NaI
−2NaBr
CH3
I
H
H
I
CH3
H
CH3
C
C
H
CH3

90.



O
O
NaHCO3
Br2
O
O
B
O
O
O
O
B